Math 153
Spring 2019

UPDATED GENERAL INFORMATION — APRIL 25, 2019

Postponement of midterm examination

The examination will take place on **Friday, May 3,** which is one class later than originally planned. It will cover all course material through Unit 4Y. Note that most of the lecture note files have been updated since the lectures in order to (1) include things mentioned in class but not in earlier versions of the documents, (2) to clarify and expand upon certain topics.

As before, about 70 per cent of the exam will involve mathematical problems, and the remaining 30 per cent will involve historical information.

MATHEMATICAL PROBLEMS. The old examinations give some idea of what to expect; these involve the use of algebra, precalculus or calculus to study issues related to the mathematics of ancient Egyptian, Babylonian and Greek civilizations. Studying the problems and solutions from previous quarter (in `exam`[*n*]`s`[*year*]`key.pdf` is strongly recommended, for some problems in this quarter's exam will be very close to items from previous quartrers. Another source of review suggestions is the document `review1-2012.pdf`. Here are still more practice problems. Solutions will be forthcoming.

1. Suppose that A, B, C, D (in that order) are the vertices of a "nice" quadrilateral which is inscribed in a circle. Then the theorem on intercepted arcs in the notes then implies that the vertex angles satisfy $|\angle ABC| + |\angle ADC| = 180^\circ = |\angle BAD| + |\angle BCD|$. On the other hand, the vertex angles of a parallelogram satisfy both $|\angle ABC| = |\angle ADC|$ and $|\angle BCD| = |\angle BAD|$. Using these, prove that if a parallelogram in inscribed in a circle, then it is a rectangle.

2. This exercise verifies a special case of Pasch's Theorem, which was also a tacit assumption in Euclid's *Elements*: Consider the triangle in the coordinate plane whose vertices are $A = (2, 0)$, $B = (0, 2)$ and $C = (-3, 0)$, let D be the external point $(0, -1)$, and let E be a point on the open segment (AC), so that $E = (q, 0)$ where $-3 < q < 2$. Show that the line DE either contains a point on closed segment $[BA]$ or a point on the closed segment $[BC]$. More precisely, show that DE contains a point of $[BA]$ if $q \geq 0$ and it contains a point of $[BC]$ if $q \leq 0$. [*Hint:* Find the points where DE meets BA and BC. Line DE meets the segment $[BA]$ if the intersection point of DE and BA has a positive second coordinate, and likewise if BA is replaced by BC.

3. Let $c > 0$ be a real number. For which values of c do the circles $x^2 + y^2 = 1$ and $(x - c)^2 + y^2 = 4$ meet in 0, 1 and 2 points? In each case, find all the intersection points.

4. Suppose that $\triangle ABC$ has a 72 degree angle at A. There are at exactly two choices of x such that $|\angle ABC| = x^\circ$ and $\triangle ABC$ isosceles. Find these values of x.

5. Suppose that we are given a right triangle in the coordinate plane with vertices $A = (0,3)$, $B = (0,0)$ and $C = (4,0)$, and let $D = (0,a)$ and $E = (b,0)$ be points on the two legs of the right triangle with $0 < a < 3$ and $0 < b < 4$. Prove that DE does not contain a point of the hypotenuse $[AC]$. It is probably best to use the intercept formula for the equation of a line joining the points $(p,0)$ and $(0,q)$:

$$\frac{x}{p} + \frac{y}{q} = 1$$

Note that a point (u,v) lies on the hypotenuse if and only if it lies on the line AC and $0 \leq v \leq 3$.

HISTORICAL QUESTIONS. These will involve major mathematical contributions or advances in various cultures and some basic knowledge of the time sequence of important developments. For example, such a question might be to state which happened first: The discovery that $\sqrt{2}$ is irrational or Plato's principle stating that an unmarked straightedege and compass were the ideal tools for geometric constructions. All of the discoveries and developments will be taken from the notes in the online directory up to the coverage limit, but some may have occurred after the era of Greek mathematics.