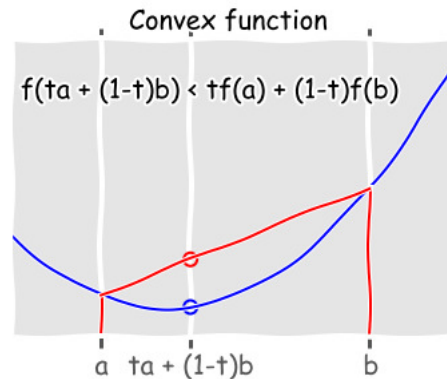


1.D. Convexity and second derivatives

A function f of one variable is said to be **convex** if for all x, y (where the function is defined) and all t between 0 and 1 we have

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y).$$

Geometrically, this means that the closed line segment joining the graph points $(x, f(x))$ and $(y, f(y))$ lies above the graph of f between x and y . The two curves may meet, but the line segment never crosses the graph. Strict convexity means that the inequality is strict.



Source: <http://people.duke.edu/~ccc14/sta-663/BlackBoxOptimization.html#optimization-of-graph-node-placement>

If the function f has a continuous second derivative, then a fundamental result about convex functions states that f is convex on an interval if and only if its second derivative is nonnegative everywhere on the interval, and it is strictly convex if and only if the second derivative is positive everywhere on the interval. A proof of this result may be extracted from the following document:

<http://math.ucr.edu/~res/math153-2019/convexity.pdf>