MORE EXERCISES RELATED TO history04*.pdf, * = X,Y,Z

11. Prove the following calculus identities for the chord function discussed in the notes and preceding exercises:

$$\frac{d}{dx}\operatorname{crd} x = \sqrt{1 - \operatorname{crd}^2 x} , \frac{d^2}{dx^2}\operatorname{crd} x = -\frac{1}{4}\operatorname{crd} x$$

12. If we are give a circular wedge with radius r and angle opening θ (measured in radians), then a standard formular states that the area of the region is equal to $\frac{1}{2}r^2\theta$. Prove this formula when $0 < \theta < \frac{1}{2}\pi$ using the setup in the file sector-area.pdf together with the following formula from a table of indefinite integrals:

$$\int \sqrt{1 - x^2} \, dx = x\sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$