Math 153
Spring 2010
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EXERCISES RELATED TO `history12.pdf` AND `history14.pdf`

As in the earlier exercises, "Burton" refers to the Seventh Edition of the course text by Burton (the page numbers for the Sixth Edition may be off slightly).

- Burton, p. 380: 11
- Burton, p. 408: 1, 3, 5, 11
- Burton, p. 432: 1, 3, 5, 11

Additional exercises

1. Although the function

$$f(x) \quad = \quad \int_0^x \frac{dt}{\sqrt{1-t^4}} \qquad (|x| < 1)$$

cannot be expressed in a finite closed form in terms of the standard functions from single variable calculus, it can be described by a power series expansion

$$\sum_{k=1}^{\infty} c_k x^k$$

by integrating the Newton binomial series for $(1-x^4)^{1/2}$. Find an explicit formula for the coefficients c_k.

2. Starting with the function $g(x)$ described at the end of `history14d.pdf`, describe infinitely differential functions $p(x)$ and $q(x)$, each defined for all real values of x, with the following properties:

(a) The function $p(x)$ is positive for all x such that $0 < x < 1$ and is zero for all other values of x.

(b) The function $q(x)$ is equal to zero for $x \leq 0$, is strictly increasing for $0 \leq x \leq 1$, and is equal to one for $x \geq 1$.

[*Hints:* Why is $g(1-x)$ infinitely differentiable, positive for $x < 1$, and zero for $x > 1$? How can one define q such that q' is a positive multiple of p?]

3. When working with trigonometric functions it is sometimes useful to know that linear combinations of the sine and cosine functions

$$a \sin\theta \ + \ b\cos\theta \ , \qquad (a, b \quad \text{real numbers})$$

are equivalent to expressions of the form $C\sin(\theta+\delta)$, where C (the *amplitude*) is nonnegative and δ is some (contstant) phase angle between 0 and 2π (with $0, 2\pi$ valid options). The standard identity for $\sin(\alpha+\beta)$ shows that expressions of the second type are equal to linear combinations of $\sin\theta$ and $\cos\theta$. Prove that, conversely, every linear combination as above can be rewritten in the second form for suitable C and θ. [*Hint:* We might as well assume $(a, b) \neq (0, 0)$ since we can take $C = 0$ and δ to be anything in that case. Let $C = \sqrt{a^2+b^2}$ so that $C > 0$, and note that we can solve the equations

$$\cos\delta \quad = \quad \frac{a}{C} \ , \qquad \sin\delta \quad = \quad \frac{b}{C}$$

for δ.]