Math 153
Spring 2010
R. Schultz

SOLUTIONS TO EXERCISES FROM `math153exercises14.pdf`

As usual, "Burton" refers to the Seventh Edition of the course text by Burton (the page numbers for the Sixth Edition may be off slightly).

Problems from Burton, p. 380

11. Use the formula in the hint. We know that $r'(\theta) = ac\exp(c\theta)$ so that the integrand is $a\exp(c\theta)\cdot\sqrt{1+c^2}$, and the result follows because

$$\int_{-\infty}^{\theta_1} \exp(c\theta)\,d\theta$$

is a convergent improper integral.

Problems from Burton, p. 380

2. Follow the hint. The formula for the tangent of the sum of two angles implies that if $\tan\alpha = \frac{1}{5}$ then

$$\tan 2\alpha = \frac{5}{12}\,, \qquad \tan 4\alpha = \frac{119}{120}\,, \qquad \tan\left(4\alpha - \tfrac{1}{4}\pi\right) = \frac{1}{239}\,.$$

Therefore if $\tan\beta = 1/239$, then $4\alpha - \frac{1}{4}\pi = \beta$ or equivalently $\frac{1}{4}\pi = 4\alpha - \beta$, which translates to the formula in the exercises.

SOLUTIONS TO ADDITIONAL EXERCISES

1. The integrand $\left(1-x^4\right)^{-1/2}$ is represented by the convergent power series from the Newton expansion for $(1-x)^a$ when $a = \frac{1}{2}$

$$1 + \sum_{k\geq 1} \frac{1\cdot 3\cdots 2k-1}{2^k\,k!}\,x^{4k}$$

so that term by term integration yields the formula

$$x + \sum_{k\geq 1} \frac{1\cdot 3\cdots 2k-1}{(4k+1)\cdot 2^k\cdot k!}\,x^{4k+1}$$

for the given antiderivative of $1/\sqrt{1-x^4}$ whose value at zero is zero.

2. (*a*) Let $p(x) = g(x)\cdot g(1-x)$.

(*b*) Let A be the integral of $p(x)$ from $x=0$ to $x=1$, and let

$$q(x) = \frac{1}{A}\cdot\int_0^x p(t)\,dt\,.$$