Math 153
Spring 2020

Head Comments on Examination 1

This is a discussion of student answers to questions on the exam and the solutions in the course directory file: `http://math.ucr.edu/~res/math153-2020/exam1s20key.pdf`

1. One common issue was the lack of a response to the part of the problem asking for reasons why different expansions of length M yield different expansions of length $M+1$. In fact, for the second part it was necessary to do slightly more; namely, to show that the expansions in the first and second part were different if the final denominator was less than $\frac{1}{4}$. Both of these points are covered in the solutions file cited above.

Here is how to prove the statement at the end of the solution, which is that the number of length M expressions grows more or less exponentially with M. Let's assume $M \geq 3$ so that the smallest unit fraction in the expression is $\frac{1}{4}$. If $E(q, M)$ is the number of length M unit fraction expansions for q, then the first part yields $E(q, M) \leq E(q, M+1) \leq E(q, M+2) \leq \cdots$. On the other hand, since $k^2 + k$ is always even we see that for each expansion of length $M+1$ one obtains two distinct expansions of length $M+2$. In other words, if $E(q, M) > 0$ then $E(q, M+2) \geq 2E(q, M)$. Hence the number of expansions at least doubles if one increases the length by 2. Since the Greedy Algorithm implies that $E(q, M) > 0$ for some choice of M, this implies that $E(q, M)$ is nondecreasing in M and $E(q, M+2k) \geq 2^k E(q, M)$ for M sufficiently large.

2. Mr. Overduin pointed out that one assertion in the second part is not necessarily correct; namely, one of the numbers dm (where $m \geq 2$ and d properly divides 40) might be equal to 40. For example, this happens if $m = 2$ and $d = 20$. The solution should have given 1 as a proper divisor of $40m$ instead. However, if we discard the next to last sentence then the remaining argument is valid as it stands.

3. One frequent omission was the lack of a proof that u satisfied one of the inequality relations involving a and c. Although the solution is based on a chain of inequalities, it also would have been enough to proceed as follows: If $a = c$ then we have a vertical line $x =$ constant, so that all three of a, u, c must be equal to that constant. Suppose now that $a < c$. Then $b < 0 < d$ means that the line $y - mx + K$ joining (a, b) and (c, d) has a positive slope m. This means that y is a strictly increasing function of x. Since the line passes through the preceding two points and $(u, 0)$, the inequality $b < 0 < d$ shows that we must have $a < u < c$. Finally, suppose that $a > c$. Then the line has a negative slope, and for the analogous reasons it follows that $a > u > c$. Partial credit was given for drawings which depicted this (as in the solutions) but were not accompanied by some sort of argument.

4. Nearly everyone received full credit for this, problem, which is essentially an exercise from a first year course in integral calculus.

5. (a) The text and the notes had conflicting years for Hippocrates of Chios (the book's numbers might be a mistaken reference to Hippocrates of Kos, the so-called Father of Medicine). All of these dates are likely to be inaccurate, but the general time periods are not, and Hippocrates of

Chios lived before Manaechmus, who is apparently the first person to construct the cube root of 2 using intersecting parabolas.

(b) The problem was meant to ask for evidence of change **within** each of these civilizations rather than their overall contributions, and the solution given in `exam1s20key.pdf` reflicts this intent, but the general interpretation in student responses was to cite contributions to all of mathematics. Since the wording was ambiguous, *such an interpretation is clearly justifiable*, so credit was given for answers of this sort. With hindsight it probably would have been better to ask for **unique** contributions from each culture, in which case one could mention the Egyptian introduction of unit fractions, the highly effective base 60 computational system of the Babylonians, and the Greek use of logic as a basic framework for their mathematical studies. In any case, all three civilizations had to be mentioned for full credit.

(c) Once again, all three civilizations had to be mentioned for full credit.

(d) Some **specific** comments were needed for full credit. For example, this could be some explanation of what sorts of constructions or underlying ideas appear at one or more key steps in the process.

To repeat a previous statement, let me know if you want more details on the specifics of the grading for your individual exam. Since all three of us participated in this grading, it might take some time to get back to you, but I will do what I can to ensure that you receive feedback in time to study for the final take-home examination.