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SOLUTION TO PROBLEM 4 WITH "CORRECT" DRAWING INTERPRETATION

Let $(x, \pm Y)$ be the points where the parabola meets the vertical line $x = h$. Then $Y^2 = 4ph$ and the volume of the cone is

$$V_{\mathrm{cone}} \quad = \quad \frac{1}{3} \cdot \pi Y^2 h \quad = \quad \frac{4p\pi\, h^2}{3} \ .$$

Furthermore, if we use the disk method to compute the volume for the paraboloid of revolution we obtain

$$V_{\mathrm{paraboloid}} \quad = \quad \int_0^h \pi\, y^2\, dx \quad = \quad \int_0^h \pi\, (4px)\, dx \quad =$$

$$4\pi\, p \left[\frac{x^2}{2} \right]_0^h \quad = \quad 2p\pi\, h^2 \ .$$

Taking ratio of the first volume to the second and simplifying, we find that this ratio is equal to 2/3.∎