Math 153
Spring 2010
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EXERCISES RELATED TO `history01.pdf`

In this list, "Burton" refers to the Seventh Edition of the course text by Burton (the page numbers for the Sixth Edition may be off slightly).

- Burton, Section 1.3, p. 28: 3, 4, 5
- Burton, Section 2.3, p. 50: 8, 13
- Burton, Section 2.4, p. 62: 9
- Burton, Section 2.5, p. 71: 4, 6, 13*abc*

Additional exercises

0. For each of the following base ten numbers, the hexadecimal form either is or looks like a word; we are using the usual convention with A = 10, B = 11, C = 12, D = 13, E = 14, and F = 15. Find the word in each case.

$$57069\ ,\quad 51966\ ,\quad 61453\ ,\quad 3499\ ,\quad 3071\ ,\quad 4013$$

There are four characters in each of the first three examples and three in each of the remaining three examples.

1. The Dozenal Societies of the U.S. and U.K. have long advocated switching our number base from ten to twelve; the extra symbols for ten and eleven are sometimes taken to be "$*$" (asterisk) for ten and "#" (hash mark/pound sign) for eleven. Convert the following base ten numbers to base twelve using this convention.

$$165, \qquad 343, \qquad 666, \qquad 998$$

NOTE. The Wikipedia article "Duodecimal" mentions examples of base twelve numeration systems in some cultures, and it also discusses modern efforts by the Dozenal Societies and others to advocate adoption of base twelve numeration (although it is highly unlikely this will ever happen, it is somewhat enlightening to think about the potential consequences).

2. Express the ordinary fractions

$$\tfrac{2}{9}\ ,\quad \tfrac{1}{25}\ ,\quad \tfrac{1}{100}\ ,\quad \tfrac{1}{125}$$

in sexagesimal form.

3. The number 1;24,51,10 appears on a Babylonian cuneiform tablet. Express it in more familiar terms.

4. Prove the assertion in the notes that a rational number r satisfying $0 < r < 1$ has only finitely many Egyptian fraction expansions with a fixed length $L > 1$. [*Hint:* Proceed by induction on the length. What happens if $L = 1$? Suppose that the result is known for length L and proceed to length $L + 1$. If one has an Egyptian fraction expansion of lemgth $L + 1$, why must one of the summands be greater than $r/(L+1)$? Show this implies that the denominator of at least one summand is $\leq (L+1)/r$. For each positive integer $m < (L+1)/r$ why do the induction hypothesis and the condition $0 < \frac{1}{m} < r$ imply that the fraction $r - \frac{1}{m}$ has only finitely many Egyptian fraction expansions of length L? How can one conclude the proof using this information?]

5. Express $\frac{p}{11}$ as an Egyptian fraction for each p such that $2 \leq p \leq 10$.

6. Babylonian mathematicians were able to find solutions to some cubic (third degree) polynomial equations in one variable by constructing tables of values for the function $y^3 + y^2$. This exercise deals with some aspects of their procedures for doing so.

(*a*) Let c be a positive real number. Prove that the equation $y^3+y^2 = c$ has only one **positive** real solution. [*Hint:* If $0 < u < v$, why is $0 < u^3+u^2 < v^3+v^2$?]. — This means that for a given choice of c there is only one admissible possibility for y; recall that the Babylonians ignored solutions that were not positive numbers.

(*b*) Using a change of variables of the form $x = ky$ for a suitably chosen positive constant k, show that one can rewrite a cubic equation of the form $x^3 + bx^2 = c$ (where b and c are both positive) in the form $y^3 + y^2 = c'$ for some $c' > 0$.

(*c*) Using a change of variables of the form $y = x + a$, show that one can rewrite a cubic equation of the form $x^3+bx^2+cx+d = 0$ in the form $y^3+b'y^2+d' = 0$. Note that in this part of the problem we make no assumptions whether or not any of the numbers b, c, d, a are positive.

7. Exercise 5 on page 80 of Burton mentions an incorrect Bablyonian formula for the area of an isosceles trapezoid $ABCD$:

$$\text{area} \quad = \quad \frac{(a+c)\cdot(b+d)}{4}$$

Here a and c are the lengths of the two parallel sides and b and d are the lengths of the nonparallel sides. The file `trapezoidABCD.pdf` gives an illustration that is consistent with Exercise 9 on page 62 of Burton. Using the formula in the exercise on page 62, find the actual area of an isosceles trapezoid in term of the lengths of the sides and trigonometric functions of the angle θ at the vertex A. Recall that the nonparallel sides have equal length, the measures of the vertex angles at A and B are equal, and the measures of the vertex angles at C and D are supplementary to those at A and B (recall that two angles are supplementary if their measures add up to $180°$). What is the ratio of the actual area to the figure given by the incorrect formula if the vertex angles at A and B are $60°$ angles?