

NAME: \_\_\_\_\_

**Mathematics 153, Spring 2005, Examination 1**

*Point values are indicated in brackets.*

1. [15 points] The sum of unit fractions

$$r = \frac{1}{7} + \frac{1}{42}$$

can also be written in the form

$$\frac{1}{n} + \frac{1}{2n}$$

for some value of  $n$ . Find the two unit fractions in this expression for  $r$ .

**SOLUTION.**

We need to solve the equation

$$\frac{1}{n} + \frac{1}{2n} = \frac{1}{7} + \frac{1}{42}$$

and if we simplify both sides of the equation this reduces to

$$\frac{3}{2n} = \frac{1}{6}.$$

This equation is equivalent to  $2n = 18$ , which means  $x = 9$ . To complete the problem we have to substitute this back into the original equation to show that

$$\frac{1}{18} + \frac{1}{9} = \frac{1}{6}. \blacksquare$$

2. [25 points] Given an odd prime  $p$  prove directly that  $6p$  is never a perfect number. As noted in class, it will suffice to do this when  $p \neq 3$ .

**SOLUTION.**

The proper divisors of  $6p$  are 1, 2, 3, 6,  $p$ ,  $2p$  and  $3p$ . These add up to  $6p + 12$ , which is greater than  $6p$ , and therefore  $6p$  is not a perfect number.■

3. [15 points] Find a positive integer  $b$  such that  $7^2 + b^2 = (b+1)^2$ . (What is  $(b+1)^2 - b^2$ ?)

**SOLUTION.**

The key to doing this is to note that  $b^2 + 2b + 1 = (b+1)^2$ . In order to solve this equation we need to find the value of  $b$  such that  $2b + 1 = 49 = 7^2$ . Solving this yields  $b = 24$ , and therefore we have the equation  $24^2 + 7^2 = 25^2$ . ■

4. [20 points] For each of the following mathematicians, identify the time during which they worked using the following key: A = before 700 B.C.E., B = between 700 B.C.E. and 500 B.C.E., C = between 500 B.C.E. and 400 B.C.E., D = between 400 B.C.E. and the death of Alexander the Great in 323 B.C.E., E = between the death of Alexander the Great and 200 B.C.E., F = after 200 B.C.E. A given letter may be the correct answer more than once.

Euclid ANSWER: E

Eudoxus ANSWER: D

Proclus ANSWER: F

Thales ANSWER: B

Zeno ANSWER: C

5. [25 points] For each of the following mathematical developments, identify when they took place using the following key: A = before 700 B.C.E., B = between 700 B.C.E. and 500 B.C.E., C = between 500 B.C.E. and 400 B.C.E., D = between 400 B.C.E. and the death of Alexander the Great in 323 B.C.E., E = between the death of Alexander the Great and 200 B.C.E., F = after 200 B.C.E. A given letter may be the correct answer more than once.

Condition of Eudoxus used to work with irrational numbers. ANSWER: D

Deductive proofs introduced into mathematics. ANSWER: B

Extensive results on Pythagorean triples known. ANSWER: A

Hippocrates' results on areas of lunes (crescent shapes). ANSWER: C

Organization of basic Greek mathematics in the *Elements*. ANSWER: E

Quadratrix of Hippias first used to solve construction problems. ANSWER: C