Math 153
Spring 2020

ADDENDUM TO EXAMINATION 2, PROBLEM 6

The first item is a typographical correction. In the first part, the radius of the locus should be $\frac{1}{2}(r+s)$.

Second, please disregard the second part. Full credit will be awarded for correctly answering the first part.

Finally, here is a list of suggested steps for solving the first part. If a point is on either circle, then it cannot be equidistant from the two circles (the distance to one is zero, the distance to the other is positive), so let's assume we are looking at points which are on neither circle. The **first part** is to show that if a point is equidistant, then it lies on the circle.

(0) Choose a coordinate system so that $\mathbf{0} = (0,0)$ is the center of both circles.

(1) Why are polar coordinates (ρ, θ) for points on the smaller circle given by all ordered pairs (s, θ) where θ runs through all real numbers and similarly the points on the larger circle given by all ordered pairs (r, θ) where θ runs trough all real numbers?

(2) Given a point P with polar coordinates (u, α) where $u > 0$, why does the preceding exercise imply that the points on the circle which are closest to P have polar coordinates (s, α) and (t, α), and what does this imply for the distances between P and the two circles? Note that there are several cases depending upon which of the statements $0 < u < s < t$, $0 < s < u < t$ or $0 < s < t < u$ is true.

(3) Using the preceding division into cases, prove that if P is equidistant from the two circles, then $0 < s < u < t$ and in fact $u = \frac{1}{2}(r+s)$. [*Hint:* What is the distance between two points with polar coordinates (X, α) and (Y, α) where the second coordinates are equal and the first coordinates are positive?]

(4) Why does this conclude the first half of the proof?

The **second part** is to show that if a point P lies on the given circle of radius $\frac{1}{2}(r+s)$, then it is equidistant from the original two circles.

(5) Show that the distance between $P = \left(\frac{1}{2}(r+s), \alpha\right)$ and each of the two circles is equal to $\frac{1}{2}(r-s)$. [*Hint:* What points on the two circles are closest to P? Why do we know this is true?]