Math 153
Spring 2012
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Preparation suggestions for the second examination

The second examination will be about 75 per cent problems and 25 per cent historical or short answer with extra credit possible on this portion of the exam. This exam will cover the material beginning with the second half of the file `history04Y.pdf` (starting on page 7 with the summary of Apollonius' work) and continuing through `history07.pdf`, and the coverage also includes the corresponding files of exercises and solutions for these units. Some of the supplementary files especially worth reviewing are `history04c.pdf`, `history05a.pdf`, `history05e.pdf`, `history06b.pdf`, `history06c.pdf`, `history06d.pdf`, `history07b.pdf` beginning with "An Alternate Approach" on page 5, `confracs.pdf`, `confracs0.pdf`, `diophantus-examples.pdf`, `fibonacci.pdf`, `fibonacci2.pdf`, `pappus-summary.pdf`, and `pappusproblems.pdf`.

For the historical part, one main thing is to know the sequence of important developments and of important figures listed in the notes, basically in order and accurate to about a century or slightly less. There will also be some questions involving comparing the works of two or more mathematicians. A review summary of the historical figures is included at the end of this document.

For the mathematical part, the problems in the exercises and the old examinations (see the subdirectory `aabOldExams` in the course directory) are good practice material. Also, a thorough understanding of background material like elementary algebra and geometry, precalculus, and first year calculus will be assumed, and problems drawing upon such knowledge are likely to be on the examination. Some aspects have been mentioned explicitly in the course notes, but a few others may appear. Here are a few additional problems worth working as preparation:

1. It is possible that there will be a problem involving continued fraction expansions for rational numbers.

2. There will probably be a problem involving Diophantine equations as worked out in the solutions to homework or `diophantus.pdf`.

3. Something related to Pappus' Centroid Theorem (`history05e.pdf` and `pappusproblems.pdf`) may appear.

4. Something related to Pell's Equation and generating new solutions from old ones (`history06b.pdf`) may appear.

5. There may be something related to the historical development of trigonometry, probably involving the function $\operatorname{crd} x$ studied by Claudius Ptolemy (know the definition!). A sample problem is given below.

6. There will probably be something involving the relationship between Pythagorean triples ($x^2 + y^2 = z^2$) and squares in arithmetic progression ($c^2 - b^2 = b^2 - a^2$) and the formulas for obtaining a triple a, b, c from a triple x, y, z (and vice versa).

Here is a problem on the Ptolemaic trigonometric function $\operatorname{crd} x$:

Prove the following double angle formula:

$$\operatorname{crd} 2\theta \quad = \quad \operatorname{crd}(\theta) \cdot \operatorname{crd}(\pi - \theta)$$

Solution. Use the description of the chord function in the notes:

$$\operatorname{crd} \alpha \quad = \quad 2\sin(\alpha/2) \ .$$

If we do this we find that the right hand side of the display is equal to

$$2\sin(\theta/2) \cdot 2\sin((\pi - \theta)/2) \quad = \quad 4\sin(\theta/2)\ \sin\big((\pi/2) - (\theta/2)\big) \quad =$$

$$4\sin(\theta/2)\ \cos(\theta/2) \quad = \quad 2\sin\theta$$

and by the chord formula this is equal to $\operatorname{crd} 2\theta$.

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Historical summary

Starred names are from the period covered on the first exam

(624 BCE - 548 BCE) Thales* — First historic figure, results in geometry.

(580 BCE - 500 BCE) Pythagoras* — Early and influential figure in development of mathematics, basic number-theoretic questions and some geometry.

(520 BCE - 460 BCE) Panini — Work on formal rules of grammar which foreshadowed 20^{th} century research on computer languages.

(490 BCE - 430 BCE) Zeno* — Formulated paradoxes which had a major impact on the subject.

(470 BCE - 410 BCE) Hippocrates of Chios* — Computed areas, wrote early but lost books on mathematics.

(460 BCE - 400 BCE) Hippias* — Quadratrix or trisectrix curve, good for trisection and circle squaring.

(428 BCE - 348 BCE) Plato* — Influential ideas about how mathematics should be studied.

(417 BCE - 369 BCE) Theaetetus* — Proof that all integral square roots of nonsquares are irrational.

(408 BCE - 335 BCE) Eudoxus* — Proportion theory for irrationals, method of exhaustion to derive formulas.

(384 BCE - 322 BCE) Aristotle* — Influential work on logic and its role in mathematics.

(380 BCE - 320 BCE) Menaechmus* — Early work on conics, duplication of cube using intersecting parabolas.

(350 BCE - 290 BCE) Eudymus* — Lost writings on the the history of Greek mathematics.

(325 BCE - 265 BCE) Euclid* — Organized fundamental mathematical material in the *Elements*, including material on geometry, number theory and irrational quantities.

(300 BCE $\pm$ 2 centuries) Pingala — Writings on language contained substantial mathematical information, including binary numeration, reference to Fibonacci sequence, results on combinatorial (counting) problems.

(287 BCE - 212 BCE) Archimedes* — Computations of areas and volumes, study of spiral curve, methods for expressing very large numbers.

(310 BCE - 230 BCE) Aristarchus — Heliocentric universe, astronomical measurements, simple continued fractions.

(280 BCE - 220 BCE) Conon* — Associate of Archimedes also associated with the Archimedean spiral

(276 BCE - 197 BCE) Eratosthenes — Prime number sieve, earth measurements.

(262 BCE - 190 BCE) Apollonius* — Extensive work on properties of conic sections, use of epicycles.

(240 BCE - 180 BCE) Diocles — Focal properties of conics.

(190 BCE - 120 BCE) Hipparchus — Early work on trigonometry, use of latter in astronomy, results in spherical geometry.

(80 BCE - 25 BCE) Vitruvius — Applications of geometry to architectural design.

(10 AD - 75) Heron — Area of triangle expressed in terms of sides.

(60 - 120) Nicomachus — Special curves, nongeometric treatment of arithmetic.

(70 - 130) Menelaus — Spherical geometry.

(85 - 165) Claudius Ptolemy — Trigonometric computations, astronomy.

(200 - 284 conjecturally) Diophantus — Algebraic equations over the integers and rational numbers, shorthand (syncopated) notation for expressing algebraic concepts.

(220 - 280) Liu Hui — Commentary on the classic Chinese *Nine Chapters on the Mathematical Art*, which was probably written during the 1^{st} century BCE, measurement results and techniques anticipating integral calculus.

(335 - 395) Theon — Influential editing of the *Elements*, commentaries.

(370 - 418) Hypatia — Daughter of Theon, lost commentaries and writings on numerous subjects.

(400 - 460 conjecturally) Sun Zi — Influential mathematical manual, containing first known problem involving the Chinese Remainder Theorem.

(410 - 485) Proclus — Commentaries on earlier Greek mathematics and its history.

(475 - 524) Boëthius — Commentaries and summaries of Greek mathematics that were widely used for many centuries.

(476 - 550) Aryabhata — Base ten numbering system mentioned in his work, introduction of trigonometric sine function, more extensive and accurate tables of trigonometric functions.

(480 - 540) Eutocius — Commentaries publicizing the work of Archimedes.

(598 - 670) Brahmagupta — Base ten numbering system explicit, free use of negative and irrational numbers, zero concept included, work on quadratic number theoretic equations over the integers, some shorthand notation employed.

(790 - 850) al-Khwarizmi — Influential work on solving equations, mainly quadratics, beginning of algebra as a subject studied for its own sake.

(800 - 870) Mahavira — Arithmetic manipulations with zero, clarification of earlier work in Indian mathematics.

(836 - 901) Thabit ibn Qurra — Original contribution to theory of amicable number pairs, extensive work translating Greek texts to Arabic.

(850 - 930) Abu Kamil — Further development of algebra.

(850 - 930) Al-Battani — Work in computational trigonometry and trigonometric identities.

(940 - 998) Al-Kuhi — Generalized version of the compass for constructing conics other than the circle.

(940 - 998) Abu'l-Wafa — Highly improved trigonometric computations, discussion of the mathematical theory or repeating geometric designs.

(950 - 1009) Ibn Yunus — Trigonometric computations and identities.

(953 - 1029) Al-Karaji/Al-Karkhi — Introduction of higher positive integer exponents and negative exponents, manipulations of polynomials, recursive proofs of formulas that anticipate the modern concept of mathematical induction.

(965 - 1040) Al-Hazen — Groundbreaking experimental and theoretical research on optics and related mathematical issues.

(1048 - 1122) Khayyam — Graphical solutions of cubic equations using intersections of circles and other conics, foundations of Euclidean geometry.

(1114 - 1185) Bhaskara — Extremely extensive and deep work on number theoretic questions including solutions to certain quadratic equations over the integers.

(1130 - 1180) Al-Samawal — Further work on polynomials, recursive proofs, formulation of the identity $x^0 = 1$.

(1170 – 1250) Fibonacci (Leonardo of Pisa) — Introduction of Hindu-Arabic numeration to nonacademics, work on number theory including Fibonacci sequence, problems involving sequences of perfect squares in an arithmetic progression, Pythagorean triples.

(1192 – 1279) Li Zhi — Research on algebraic equations and number theory, with applications of algebra to geometric problems.

(1201 – 1274) al-Tusi, Nasireddin — Early work on making trigonometry a subject in its own right, foundations of Euclidean geometry.

(1202 – 1261) Qin Jushao — Wrote *Mathematical Treatise in Nine Sections* which summarizes much of Chinese work in mathematics at the time and breaks new ground.

(1220 – 1280) al-Maghribi — Commentaries on the apocryphal Books XIV and XV of Euclid's *Elements*.

(1225 – 1260) Jordanus Nemoriarus — Limited use of letters, results on perfect versus non-perfect numbers, relations between spherical and plane geometry (via stereographic projection), problems related to physics.

(1238 – 1298) Yang Hui — Research on algebraic an number-theoretic questions, including magic squares.

(1260 – 1320) Zhu Shijie — Algebraic summation formulas, solutions to some higher degree polynomial equations in several unknowns.

(1323 – 1382) Oresme — Summations of certain infinite series, early ideas on the graphical representation of functions.

(1350 – 1425) Madhava — Early figure in the Kerala School of mathematics, infinite series formulas for inverse tangent and π.

(1370 – 1460) Parameshvara — Early version of the Mean Value Theorem in calculus in the Kerala school.

(1377 – 1446) Brunelleschi — First specifically mathematical study of drawing in geometric perspective.

(1380 – 1450) al-Kashi — Free use of decimal fractions and (infinite) decimal expansions, computation of π, Law of Cosines.

(1444 – 1544[*sic*]) Nilakantha Somayagi — Computation of π via infinite series in the Kerala school.

(1664 – 1739) Seki — Independent discovery of many results in calculus, number theory and matrix algebra, all done in Japan when that country was almost completely cut off from the rest of the world.