Computational Details for

Exercise V. 5.103

The underlying idea is to use the standard long division procedure modified for base 2.

$\frac{1}{3}$ three = 11 in base 2

$$
\begin{array}{r l}
 & 0.0101\ldots \quad \leftarrow \text{ANSWER} \\
11\,) & 1.00000 \\
 & 0 \\
\hline
 & 100 \\
 & 11 \\
\hline
 & 10 \\
 & 0 \\
\hline
 & 100 \\
 & 11 \\
\hline
 & 10
\end{array}
$$

periodic sequence

remainder 10

remainder 1

remainder 10

remainder 1

etc.

The long division steps repeat with remainders of 10, 1, 10, 1, ... (all base two) so the 1's and 0's also repeat.

$\frac{1}{4}$ Since $\frac{1}{4} = \frac{1}{2^2}$, we get the expansions

0.01 and 0.001111...

$\frac{1}{5}$ five = 101 in base 2

periodic sequence

```
     0.0011 0011 ...      ← ANSWER
101 ) 1.000000
      0
      10 0                remainder 10
         0
      1000                remainder 100
       101
        110               remainder 11
        101
          10              remainder 1
           0
          100             remainder 10
              etc.
```

Here the long division steps repeat with period 4 and remainders 10, 100, 11, 1 (all less than 101)

$\frac{1}{6}$ This is just $\frac{1}{2} \times \frac{1}{3}$, so by the usual rules for decimal multiplication and the result for $\frac{1}{3}$ we get 0.0010101 01 ...

periodic seq.

$\frac{1}{7}$ seven = 111 in base 2 ANSWER

$$\begin{array}{r}
0.001001001\ldots \\
111 \overline{)\,1.0000000000} \\
0 \\
\hline
100 \\
0 \\
\hline
1000 \\
111 \\
\hline
10 \\
0 \\
\hline
100
\end{array}$$

periodic piece

remainder 10

remainder 100

remainder 1

remainder 10

etc.

The long division steps repeat with period 3 and remainders 10, 100, 1.

$\frac{1}{8}$ This is $\frac{1}{2^3}$, so the binary expansion is either 0.001 or .0001111.....

$\frac{1}{9}$ nine = 1001 in base 2.

```
          0.000 111 000 111   ← ANSWER
1001 ) 1.000000000000000
        0
        10                     remainder 1
        0
        100                    remainder 10
        0
        1000                   remainder 100
        0
        10000                  remainder 1000
         1001
          1110                 remainder 111
          1001
           1010                remainder 101
           1000
              1                remainder 1
             etc.
```

periodic piece

In this case the the remainders repeat with period 6.