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Comments on posted solutions to the exercises

These solutions are posted mainly for students to compare with their own efforts and to determine whether their solutions are correct or can be improved upon. However, in some cases the solutions might also be useful if and when a student reaches an impasse. At some points, solutions to some of the more complicated exercises may be parts of reading assignments, but as a general rule **the solutions should not be viewed as an excuse for not trying to work the exercises, especially those that are specifically assigned in course announcements** (*i.e.*, the `aabUpdate`$*$ files in the course directory). Problems at the level of the assigned exercises are likely to appear on the course examinations, so it is important for students to be able to work out such solutions on their own.

Suggestions for working exercises

The directory file `polya.pdf` contains a systematic and general list of suggestions for approaching and solving mathematical problems, and the file `mathproofs.pdf` discusses the more formal aspects of mathematical proofs. Here are a few more comments:

1. Many exercises can be solved by imitating arguments in the course text or notes, so it is usually worthwhile to see if an exercise can be analyzed by modifying a previously seen argument. The following quotation from the *Poetics* of Aristotle (384–322 B.C.E.; see Section I, Part IV) seems appropriate:

> *The instinct of imitation is implanted in man from childhood, one difference between him and other animals being that he is the most imitative of living creatures, and through imitation learns his earliest lessons.*

The differences between human and animal behavior in some species are probably less than they seemed to the ancient Greeks, but the passage still reflects the importance of imitation in human thought and action.

2. The first efforts (and in many cases subsequent efforts!) at solving exercises will not necessarily be as clear or polished as the proofs and solutions in textbooks or the files posted to the course directory. Often the first attempts to find solutions are at least somewhat messy, and they usually get better as a result of increased experience, skills, and trial and error. The following quotation due to A. S. Besicovitch (1891–1970) summarizes everything in a somewhat ironic manner:

> *A mathematician's reputation rests on the number of bad proofs he has given.*

3. It is usually good to try anything that might work rather than not getting started with work on a problem. This advice reflects a frequently repeated quotation attributed to L. R. ("Yogi") Berra (1925–2015):[1]

> *When you come to a fork in the road ... Take it.*

Likewise, if a solution to a problem is not apparent after some thought, it is often worthwhile to be systematic and look at everything that can be said about the given situation, no matter how

[1] All such quotes must be viewed in light of another one: "I really didn't say everything I [supposedly] said. ... Then again, I might have said 'em, but you never know."

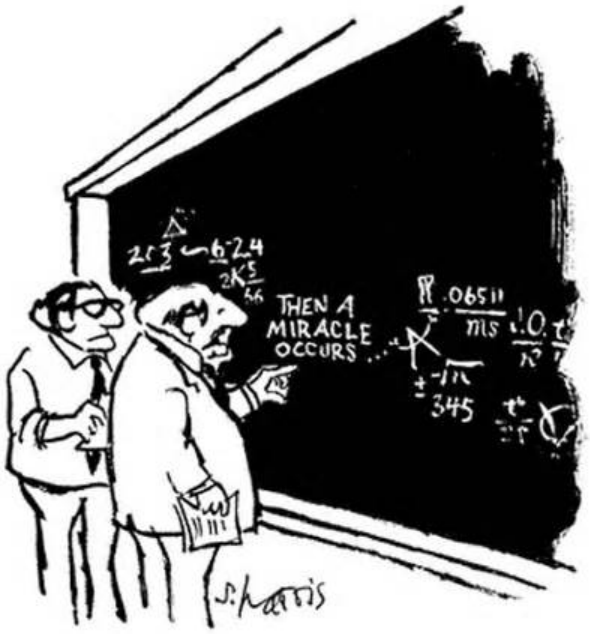
insignificant it might seem at first. This is summarized in a quotation from one of the *Sherlock Holmes* stories by A. C. Doyle (1859–1930):

> *You know my method. It is founded upon the observation of trifles.*

4. Although rigorous mathematical proofs must be expressed in words rather than pictures, in many cases good drawings are extremely helpful sources for insights which can suggest an approach to proving a mathematical statement. Probably the two most basic situations involve Venn diagrams to analyze the algebraic properties of set-theoretic operations and Hasse diagrams to visualize orderings of objects. Both types of diagrams are defined (with examples) in the Schaum's Outline Series book by Lipschutz.

5. The first step in putting together a logical argument is to come up with something that appears to be correct, but one important test of an argument's clarity and validity is to refine the ideas so that they become equally clear and convincing to someone else who is reasonably well-informed about the subject. This generally requires healthy doses of skepticism and self-criticism; these can certainly be overdone, but initially most of us need to be more concerned about not going far enough.

The next page contains two well-known satirical cartoons by Sidney Harris (1933–) on arguments that are convincing to someone else. There are several anthologies of Harris' excellent cartoons on humorous aspects of scientists and their work, and they are definitely worth reading and viewing.



"I think you should be more explicit here in step two."