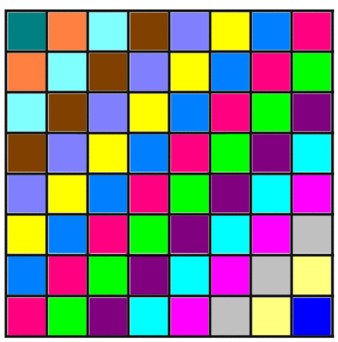
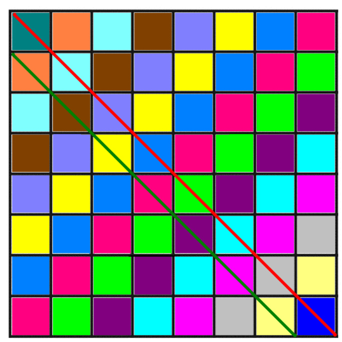
FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

IV.1: Binary relations

102. Assume we label the squares by two positive integers on the board from left to right and from the bottom to the top. The first step in the argument is to show that a bishop can move diagonally to another square of the same color. In other words, if the bishop is located at the point whose horizontal coordinate is i and whose vertical coordinate is j, then the bishop can move one square up or down, to the square whose horizontal coordinate is $i+1$ and whose vertical coordinate is $j-1$, or to the square whose horizontal coordinate is $i-1$ and whose vertical coordinate is $j+1$, provided there are such squares on the board. Thus each diagonal lies in an equivalence class of points such that a bishop can move from one square to another in the class, and since there are exactly 15 diagonals in the drawing, this means there are at most 15 equivalence classes (see the drawing on the left).



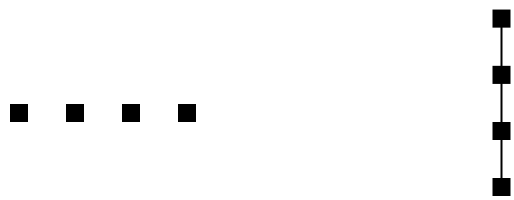


The final step in the argument is to note that the points on the red and green lines in the right hand drawing also lie in the same equivalence class. Since the two lines contain exactly one square of each color, it follows that there are at most two equivalence classes of squares, and they are distinguished by whether $i+j$ is even or odd. In fact, there are exactly two such equivalence classes, for if the bishop moves one square from the position with coordinates i and j to a square with coordinates p and q, then by construction the sums $i+j$ and $p+q$ are both even or both odd.

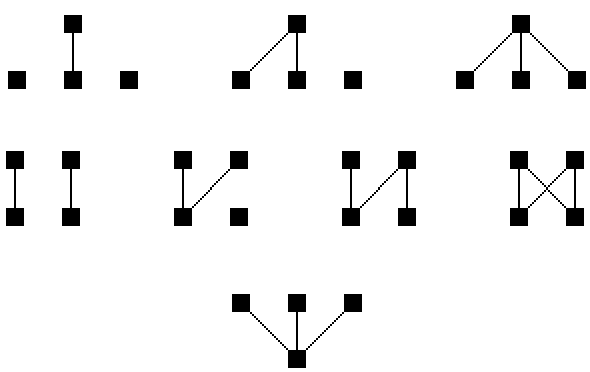
See the file **http://math.ucr.edu/~res/math144-2017/knight2014.pdf** for remarks on the squares that a knight on a chessboard can reach.

IV.6: Order types

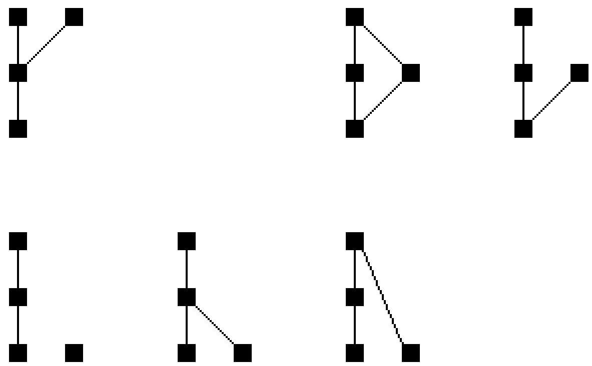
103. Here are Hasse diagrams for each of the **16** order types for which the underlying set has four elements. We shall begin with the extreme cases where the highest level L of an element is **1** or **4.** In each of these cases there is only one order type; if $L = 1$ then the partial ordering is just the equality relation (this is sometimes described as ***totally unordered***), and if $L = 4$ then the partial ordering is a linear ordering.



The next class of cases are those for which $L = 2$, so that there are elements of levels **1** and **2.** There are three subcases, depending upon whether there are **3, 2,** or **1** elements of level **1;** this means there are respectively **1, 2,** or **3** elements of level **2.** Hasse diagrams for each of the **8** cases are depicted below:



The final class of cases are those for which $L = 3$, in which there are elements of levels **1, 2,** and **3.** There are three subcases, in which the numbers of elements at these respective levels are $(1, 1, 2)$, $(1, 2, 1)$ and $(2, 1, 1)$. Hasse diagrams for each of the **6** cases are depicted on the next page:



It follows that the total number of order types for a set with $\mathbf{4}$ elements is equal to the sum of the numbers of types with $L = \mathbf{1, 2, 3, 4}$ is equal to $\mathbf{1 + 8 + 6 + 1 = 16.}$