## Some short answer questions with answers

Here are a few examples. They are intended to be similar to the short answer questions on an Advanced Placement (AP) history exam, but with shorter answers. An Internet search for AP short answer questions might also be useful practice. We shall give the problems on the first page with acceptable responses on subsequent pages. Accuracy and coherence are the main things that are needed, but of course the answers must also be reasonably legible; you might want to type them out with a word processor or text editor. If you have an alternate response you wish to check, please contact me./RS

- 1. Give a potential reason why Euclid's <u>*Elements*</u> survived but the similarly named writings of Hippocrates of Chios apparently did not.
- 2. Describe one similarity and one difference between the work of Archimedes and Apollonius.
- **3.** Give one reason we know that the conic section curves had been discovered before the time of Euclid.
- **4.** In what ways did later Greek mathematics turn to Egyptian, Babylonian or earlier Greek mathematics as it shifted directions?
- **5.** Give examples of accurate and inaccurate results due to Eratosthenes, Aristarchus and Posidonius on measuring the sizes of the earth, moon and sun and their distances from each other.
- **6.** What key property of the real numbers was crucial to the theory of irrational proportions developed by Eudoxus and Euclid?
- 7. Compare and contrast the roles of deductive logic in Egyptian, Babylonian and Greek mathematics.
- 8. Compare and contrast the mathematical impacts of the writings of Plato and Aristotle.

Before providing written solutions, here are a couple of remarks. The first is **DO NOT PANIC** when reading the questions for the first time. The answers may not be trivial, but they are not all that obscure. Second, my general expectation is that one can give acceptable answers in at most three sentences, although there will be no limit on length (but the longer the answer, the greater the risk that coherence might be lost).

## ACCEPTABLE RESPONSES

## These are not unique, and usually there are several alternative answers that are also correct. Some of these are considerably longer than what would be needed.

**Question 1.** Euclid's work appeared much later and was necessarily more complete. Another likely reason is that it treated many topics in a more definitive fashion.

**Question 2.** Both Archimedes and Apollonius studied complicated problems and discovered answers to them. In both cases their writings were usually logically rigorous. Both also studied problems arising outside of mathematics itself. On the other hand, the impact of Archimedes' work was largely on physics and engineering, although there were substantial mathematical contributions as well. In contrast, the main impact of Apollonius' work was on mathematics itself. Both studied problems that inspired later researchers over 1700 years later.

**Question 3.** In the 4<sup>th</sup> century B. C. E. Manaechmus discovered a way to duplicate the cube by means of intersecting parabolas. Also, Euclid wrote a book on conics which is now lost.

**Question 4.** As the Hellenistic period progressed, Greek mathematics became increasingly focused on questions from astronomy, and the computational techniques of the Babylonians were superior to their own. This played a role in the creation of trignometric tables. Later work of mathematicians like Heron and Nichomachus took a more algebraic approach to equations than the intrinsically geometric approach during the most productive periods in Greek mathematics. Also, Nichomachus went back to the perspectives and interests of the Pythagorean school in many respects (perfect and amicable numbers are examples, and he even took things further).

**Question 5.** The measurement of the Earth's diameter by Eratosthenes was remarkably accurate for its time, and some of the lunar measurements by Aristarchus were also quite good. Also, Aristarchus succeeded in discovering that the distance from the earth to the sun and stars was far greater than previously understood. However, Aristarchus' quantitative measurements of the sun were far off the mark. Another bad measurement with tremendous consequences was a measurement of the earth's diameter by Posidonius, which indirectly led to the (re)discovery of the Western Hemisphere by Columbus. He first requested financing from the Portuguese, who were the main maritime power of the 15<sup>th</sup> century. They turned him down because they knew from Eratorsthenes' computation that ships could not store adequate provisions for a sea voyage from Portugal to India. Unperturbed, Columbus turned to Spain and convinced them to provide financing by using the smaller estimate due to Posidonius. It is interesting to speculate what might have happened if Spain had had better informed scientific advisors.

**Question 6.** The Condition of Eudoxus and Euclid's application of it rely on the fact that a real number is completely determined by (1) the rational numbers which are greater than it, (2) the rational numbers which are less than it. These two facts were also crucial to the development of a logically rigorous theory of real numbers in the 19<sup>th</sup> century by R. Dedekind. The two properties follow from the density of the rational numbers: Given two real numbers, there is a rational number between them.

**Question 7.** All of these cultures used deductive logic, at least to some extent, to analyze and solve problems and to develop skills for applying the methods which work for one problem to a similar problem. However, the Greeks took things one step further and used logic as a framework for organizing much (most?) of their mathematical knowledge. This comment about Greek mathematics should probably be qualified by noting that they viewed most arithmetic computation as an inferior discipline.

**Question 8.** Plato devised a framework for doing mathematics which was more or less definitive for subsequent Greek work and in some respects is still basic to modern mathematics. Aristotle wrote extensively on the logic which was needed to implement Plato's framework. One less important difference is that Plato himself did not prove any mathematical results, but his students did so; one result due to Aristotle is in the homework exercises.