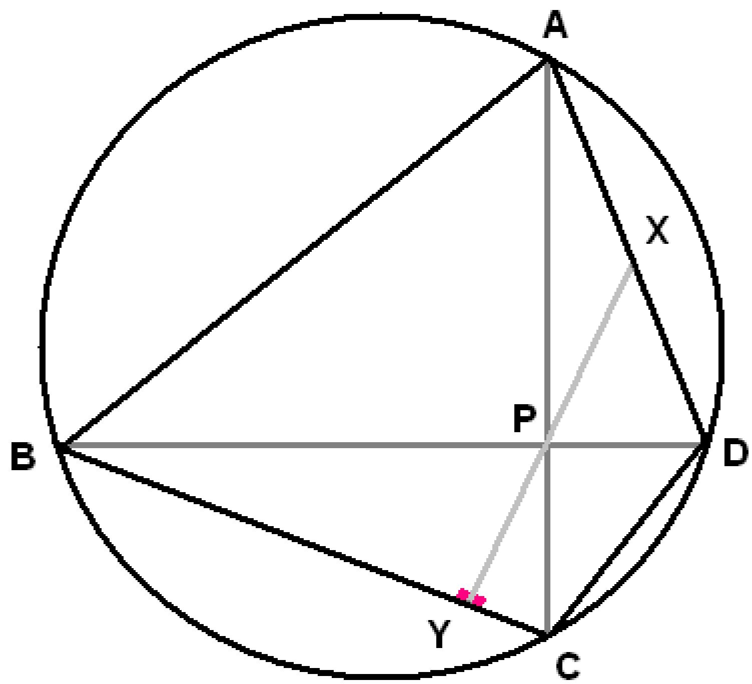
Notes problem 4.4.7 in Burton (on page 183)

Since the drawing in Burton does is slightly inaccurate (the lines **XY** and **BC** do not look perpendicular), here is a better one.



By assumption, the lines **XY** and **BC** are also perpendicular.

As noted in the solution to this problem, we have glossed over issues like whether one point is between two others or in the interior of a given angle, and our purpose here is to justify these intuitively reasonable points. In order to do this, we need to assume familiarity with the course notes in http://math.ucr.edu/~res/math133.

We are basically given that the diagonals **[AC]** and **[BD]** meet at some point **P** between the endpoints, and let us also take for granted that the foot of **Y** the perpendicular from **P** to **BC** lies on the open segment **(BC),** and the point **X** where **PY** meets **AD** lies on the open segment **(AD).**

In order to apply the result on intercepted arcs, we need to know that the intercepted arcs for the angles **BCA** and **BDA** are the same, which amounts to knowing that **C** and **D** lie on the same side of the line **AB.** But we know that **P** lies on both segments **(BD)** and **(AC),** and this is enough to imply that **C** and **D** lie on the same side of **AB** as **P.**

In order to prove that the measures of the angles **CPY** and **BPY** add up to **90** degrees, we need to know that **Y** lies in the interior of the right angle **BPC.** However, this follows since **Y** lies on the open segment **(BC),** so that **Y** and **C** are on the same side of **BP** and **Y** and **B** are on the same side of **CP.**

Similar considerations hold if we reverse the roles of **A** and **D** and of **B** and **C** in the preceding discussion.