Math 153
Spring 2020
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SOLUTIONS TO EXERCISES FROM `week7/unit06Y/homework06y.pdf`

As usual, "Burton" refers to the Seventh Edition of the course text by Burton (the page numbers for the Sixth Edition may be off slightly). This document contains solutions to assigned problems from Burton which had not previously been worked out.

Problems from Burton, p. 264

10. The goal of the problem would have been much clearer with an explanation that "root" should be interpreted as "square root." This explains the hint given in the problem, and using this hint we may proceed as follows: We want to solve for x such that $x = y^2$ and

$$20 - 3y = 4 \cdot \sqrt{y^2 - 3y} .$$

If we square both sides as indicated in the hint we obtain

$$400 - 120y + 9y^2 = 16y^2 - 48y$$

which simplifies to $7y^2 + 72y - 400 = 0$. One can use the quadratic formula to find the roots of this polynomial, and they are $y = 4$ and $y = -100/7$. For the first choice we find that $x = 16$ as in Burton. Presumably the second choice is ignored because abu-Kamil did not consider negative numbers.∎

11. The first step is to formulate the problem using mathematical symbols. We are given that $10 = x + y$ and $9 = x^2/y$. The second equation is equivalent to $9y = x^2$ (the condition $y \neq 0$ is implicit in forming the fraction x/y), and this yields the quadratic equation

$$10 = x + \frac{x^2}{9} \quad \text{or equivalently} \quad x^2 + 9x - 90 = 0 .$$

The roots of this equation are 6 and -15, and as before we ignore the negative root. This implies that $x = 6$ and $y = 4$.∎

12. Again follow the hint, writing $x = y^2/3$ and $y^2 + 21 = 10y$. It follows that $y^2 - 10y + 21 = 0$, which means that $y = 3$ or $y = 7$. In the first case we see that $x = 3$, and in the second we see that $x = 49/3$.∎

13. Following the hint and noting that x and y are implicitly assumed to be nonzero, we have a system of equations given by $x + y = 10$ and $2000 = 125xy$. The second equation reduces to $16 = xy$, and if we substitute $y = 16/x$ into the first equation we see that

$$10 = x + \frac{16}{x} \quad \text{or equivalently} \quad x^2 - 10x + 16 = 0 .$$

The roots of this equation are $x = 2$ and $x = 8$. Substituting these into $x + y = 10$, we find that $y = 8$ and $x = 2$ in these respective cases. ∎