Burton, page 326, Problem 8

<u>Note:</u> The meaning of "twice its square root" is unclear. Do they mean "it" is the original number plus twice its square root, or do they mean the original number? The hint suggests the first interpretation.

So we want to find $\underline{\underline{x^2}}$ such that

$$y^2 = x^2 + 2x \quad \text{and} \quad y^2 + 2y = 10.$$

First solve $y^2 + 2y - 10 = 0$, obtaining

$y = -1 \pm \sqrt{11}$. ~~Since y~~

To get the book's solution, take the positive root.

Then $y^2 = 12 - 2\sqrt{11}$. But now we have

$$x^2 + 2x = y^2 = 12 - 2\sqrt{11}$$

Let's solve this by completing the square

$$x^2 + 2x + 1 = (x+1)^2 = 13 - 2\sqrt{11}$$

The book now takes

$$x + 1 = \sqrt{13 - 2\sqrt{11}}\ ,$$ so that

$$x = -1 + \sqrt{13 - 2\sqrt{11}}$$ and hence

$$x^2 = \left(14 - 2\sqrt{11}\right) - 2\sqrt{13 - 2\sqrt{11}}$$

which is the answer found in Burton (see page 770).

Corrected solution to 3(f) (Burton, p 326)

The right polynomial for this problem is $x^3 - 3x^2 - 27x - 41 = 0$. If $y = x-1$, this becomes $y^3 - 30y - 70 = 0$, so that $p = -30$ and $q = 70$. This means

$$\frac{q}{2} = +35, \quad \frac{q^2}{4} = 1225, \quad \frac{p^3}{27} = \frac{(-30)^3}{3^3} = -1000$$

By the formula on p. 323 of Burton, this yields

$$x = \sqrt[3]{35 + \sqrt{1225-1000}} - \sqrt[3]{-35 + \sqrt{1225-1000}} \overset{(**)}{=}$$

$$\sqrt[3]{35+15} - \sqrt[3]{-35+15} = \sqrt[3]{50} - \sqrt[3]{-20} =$$

$$\sqrt[3]{50} + \sqrt[3]{20}.$$

(**) Note that $\sqrt{1225-1000} = \sqrt{225} = \sqrt{15}$.