Math 153
Spring 2019

UPDATED GENERAL INFORMATION — APRIL 19, 2019

The first quiz

This will take place on Thursday, April 25, and it will cover material from Units 1 and 2. Here are some practice problems. Solutions will be forthcoming.

1. Find an Egyptian fraction expression for $\frac{5}{13}$.

2. Find a sexagesimal approximation to $\frac{5}{13}$ of the form $a'b''c'''$ where a, b, c are integers between 0 and 59; in modern notation, this is a fraction of the form $a \cdot 60^{-1} + b \cdot 60^{-2} + c \cdot 60^{-3}$.

3. Prove that 45 is not a perfect number and not part of an amicable pair.

4. Same problem for p^3 if $p > 3$ is prime. [*Hints:* If d is a positive integer which evenly divides the odd integer q and $d \neq q$, why do we know that either $d \leq \sqrt{q}$ or else $q/d \leq \sqrt{q}$? Why does this yield an upper bound on the number of positive integers evenly dividing q? To estimate the sum of the proper divisors of q, combine this with the fact that if q is odd and d is a proper divisor of q, then $d \leq q/3$.]

5. The octagonal numbers satisfy the following identities for $n \geq 2$:

$$\mathrm{Oct}_2 \quad = \quad 8, \qquad \mathrm{Oct}_{n+1} = \mathrm{Oct}_n \ + \ 6n + 1$$

Prove by induction that $\mathrm{Oct}_n = 3n^2 - 2n$ for all $n \geq 2$.

6. Prove that $105p$ is not a perfect number if $p \geq 11$ is prime.