## 0.C. Some facts from algebra

A good understanding of the algebra in precalculus and calculus will be an implicit assumption for this course. For the sake of convenience, we mention two specific items which will be used repeatedly throughout the course.

Euclidean (or "long") division property. Given two positive integers $\boldsymbol{a}, \boldsymbol{b}$ with $\boldsymbol{a}<\boldsymbol{b}$, there are unique nonnegative integers $\boldsymbol{q}$ and $\boldsymbol{r}$ such that $\boldsymbol{b}=\boldsymbol{a} \boldsymbol{q}+\boldsymbol{r}$ and $\mathbf{0} \leq \boldsymbol{r}<\boldsymbol{a}$.

The number $\boldsymbol{q}$ is often called the integral quotient, and $\boldsymbol{r}$ is often called the remainder of $\boldsymbol{b}$ after division by a.

Comparing fractions or proportions. If $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ are positive real numbers, then the quotient $\boldsymbol{a} / \boldsymbol{b}$ is less than, equal to, or greater than $\boldsymbol{c} / \boldsymbol{d}$ if and only if $\boldsymbol{a d}$ is less than, equal to, or greater than $\boldsymbol{b} \boldsymbol{c}$ (respectively).

This follows because the second pair of numbers is equal to the first pair multiplied by the positive quantity $\boldsymbol{b} \boldsymbol{d}$, and multiplying by a positive number does not change statements about the equality or the direction of an inequality between two numbers.

