0.G. Notational conventions for elementary geometry

Since concepts from elementary geometry play an important role in any account of the history of mathematics, we shall frequently discuss various geometrical results. Of course, symbolic notation will make it easier to formulate many statements, but unfortunately there are no well – established notational conventions for many key concepts. Therefore we have summarized the main features of our notation for reference purposes. We begin with six conventions involving lines. Many are surely self – explanatory, but a few might seem arbitrary.

1. Given two points **A** and **B**, the unique ***line*** joining them will be denoted by **AB.**
2. Given two points **A** and **B**, the ***closed line segment*** joining them, which consists of **A, B,** and all points **X** which lie between **A** and **B**, will be denoted by **[AB];** similarly, the ***open line segment*** joining them, which consists of all points **X** which lie between **A** and **B**, will be denoted by **(AB).**
3. Given two points **A** and **B**, the ***(closed) ray starting at* A *and passing through* B**, which consists of **A, B,** all points **X** which lie between **A** and **B**, and all points **X** such that **B** lies between **A** and **X**, will be denoted by **[AB.** Equivalently, this is the set of all points **X** on the line **AB** such that **A** is <u>**NOT**</u> between **X** and **B.**
4. The statement that ***a point* X *lies between* A *and* B** will often by written symbolically as **A∗X∗B.**
5. Given two points **A** and **B**, the ***distance*** from **A** to **B**, or equivalently the ***length of the closed segment*** **[AB]**, will be denoted by **|AB|.**
6. Given three noncollinear points **A, B** and **C**, the ***triangle*** with vertices **A, B, C** — written **△ABC** — is the union of the three closed segments **[AB], [BC]** and **[AC].** This is a "hollow triangle" as opposed to the "solid triangular region" consisting of the triangle and all points which are "inside" the triangle (see page 60 of the online document http://math.ucr.edu/~res/math133/geometrynotes2b.pdf for a formal definition of a triangle's interior).

Here are some drawings for the first three items:

The drawing above depicts the line **AB.**

The drawing above depicts the closed line segment **[AB].**

The drawing above depicts the closed ray **[AB.**

In the drawing above, the statements **A∗X∗B** and **B∗X∗A** are both true, but the statements **A*B*X** and **X∗A∗B** are both false, and similarly the statements **X∗B∗A** and **B∗A∗X** are both false.

Next, we shall give the conventions involving angles.

7. Given three noncollinear points **A, B** and **C,** the ***angle*** $\angle$**ABC** is defined to be the union of the rays **[BC** and **[BA.** The ***measure*** of this angle, usually but not always expressed in degrees throughout the notes, is denoted by **|**$\angle$**ABC|.**

IMPORTANT REMARKS. This definition **excludes** the extreme concepts of a ***zero – degree angle*** for which the two rays are equal and of a ***straight angle*** in which the two rays are opposite rays on the same line (and the points in question satisfy **A$*$B$*$C**).

It follows immediately from the definitions that $\angle$**CBA** = $\angle$**ABC.** Note that ***the statement*** $\angle$**ABC** = $\angle$**DEF** ***is*** **much stronger** ***than saying the two angles have the same measures*** (in symbols, **|**$\angle$**ABC|** = **|**$\angle$**DEF|**); it means that ***the two angles consist of*** **exactly the same points**.