

The condition of Eudoxus

Two real numbers a and b are equal if and only if

(1) For every rational number q such that $a < q$ we also have $b < q$.

(2) For every rational number q such that $a > q$ we also have $b > q$.

The reason for this is simple: Given two real numbers x and y such that $x < y$ there is a rational number r such that $x < r < y$.