CONTINUED FRACTIONS

$x_0 = \frac{8}{13}$

$$y_0 = \frac{13}{8} = \underset{n_1}{1} + \underset{x_1}{\frac{5}{8}}$$

$$y_1 = \frac{8}{5} = \underset{n_2}{1} + \underset{x_2}{\frac{3}{5}}$$

$$y_2 = \frac{5}{3} + \underset{n_2}{1} + \underset{x_3}{\frac{2}{3}}$$

$$y_3 = \frac{3}{2} = \underset{n_4}{1} + \frac{1}{2} \quad x_4 = \frac{1}{n_5}$$

Therefore

$$\frac{8}{13} = \cfrac{1}{\underset{n_1}{1} + \cfrac{1}{\underset{n_2}{1} + \cfrac{1}{\underset{n_3}{1} + \cfrac{1}{\underset{n_4}{1} + \underset{n_5}{\frac{1}{2}}}}}}$$

Notice this always ends with the last integer greater than 1.

Since

$$\frac{1}{n_m} = x_{m-1} < 1$$

Inverse Problem

Given $4, 3, 2$ find x_0.
$\quad\quad\; m_1\; m_2\; m_3$

$$x_2 = \frac{1}{m_3} = \frac{1}{2}$$

$$y_2 = m_2 + x_2 = 3 + \frac{1}{2} = \frac{7}{2} \qquad x_1 = \frac{2}{7}$$

$$y_1 = m_1 + x_1 = 4 + \frac{2}{7} = \frac{30}{7} \qquad x_0 = \frac{7}{30}.$$

Abstract example

$$x_0 = \frac{m-1}{m} \qquad y_0 = \frac{m}{m-1} = \underset{m_1}{1} + \frac{1}{m-1} \qquad x_1 = \frac{1}{m_2}$$

Therefore $x_0 = \dfrac{1}{1 + \frac{1}{m-1}}$