History of Mathematics

Mathematics in China

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A brief outline of the history of Chinese mathematics

Primary sources are Mikami's *The Development of Mathematics in China and Japan* and Li Yan and Du Shiran's *Chinese Mathematics, a Concise History*. See the bibliography below.

1. Numerical notation, arithmetical computations, counting rods
   - Traditional decimal notation -- one symbol for each of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 100, 1000, and 10000. Ex. 2034 would be written with symbols for 2,1000,3,10,4, meaning 2 times 1000 plus 3 times 10 plus 4. Goes back to origins of Chinese writing.
   - Calculations performed using small bamboo counting rods. The positions of the rods gave a decimal place-value system, also written for long-term records. 0 digit was a space. Arranged left to right like Arabic numerals. Back to 400 B.C.E. or earlier.
   - Addition: the counting rods for the two numbers placed down, one number above the other. The digits added (merged) left to right with carries where needed. Subtraction similar.
   - Multiplication: multiplication table to 9 times 9 memorized. Long multiplication similar to ours with advantages due to physical rods. Long division analogous to current algorithms, but closer to "galley method."

2. *Zhoubi suanjing (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven)* (c. 100 B.C.E.-c. 100 C.E.)
   - Describes one of the theories of the heavens. Early Han dynasty (206 B.C.E -220 C.E.) or earlier. Book burning of 213 B.C.E..
   - States and uses the Pythagorean theorem for surveying, astronomy, etc. Proof of the Pythagorean theorem.
   - Calculations including with common fractions.

3. *The Nine Chapters on the Mathematical Art (Jiuzhang Suanshu)* (c. 100 B.C.E.-50 C.E.)
   Collects mathematics to beginning of Han dynasty. 246 problems in 9 chapters. Longest surviving and most influential Chinese math book. Many commentaries.
   - Ch 1, Field measurement: systematic discussion of algorithms using counting rods for common fractions including alg. for GCD, LCM; areas of plane figures, square, rectangle, triangle, trapezoid, circle, circle segment, sphere segment, annulus -- some accurate, some approximations.
   - Ch 2,3,6 on proportions, Cereals, Proportional distribution, Fair taxes.
   - Ch 4, What width?: given area or volume find sides. Describes usual algorithms for square and cube roots but takes advantage of computations with counting rods
   - Ch 5, Construction consultations: volumes of cube, rectangular parallelepiped, prism frustums, pyramid, triangular pyramid, tetrahedron, cylinder, cone, and conic frustum, sphere -- some

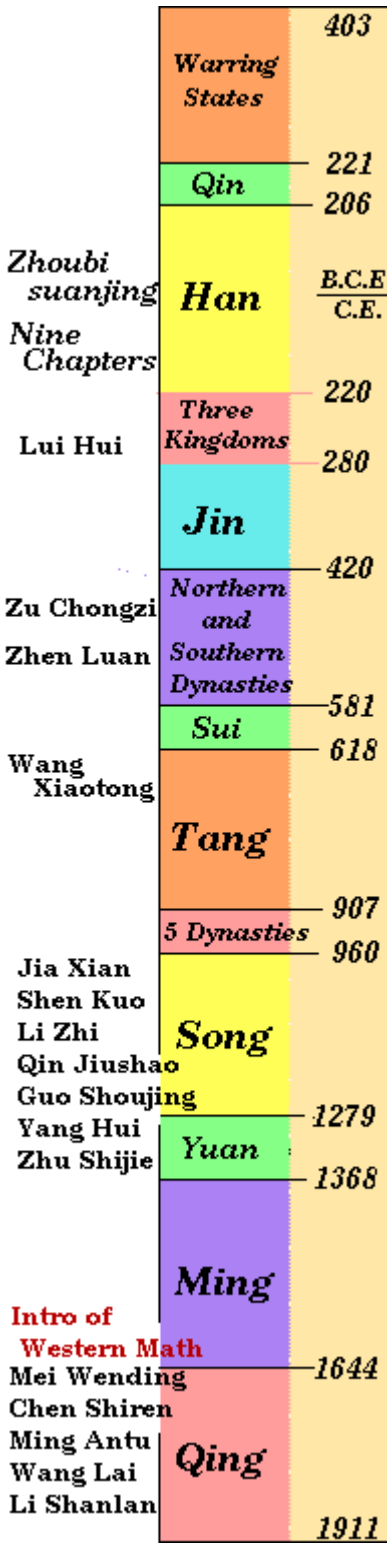
approximations, some use *pi*=3
- Ch 7, Excess and deficients: false position and double false position
- Ch 8, Rectangular arrays: Gives elimination algorithm for solving systems of three or more simultaneous linear equations. Involves use of negative numbers (red reds for pos numbers, black for neg numbers). Rules for signed numbers.
- Ch 9, Right triangles: applications of Pythagorean theorem and similar triangles, solves quadratic equations with modification of square root algorithm, only equations of the form $x^2 + a\,x = b$, with a and b positive.

4. Sun Zi (c. 250? C.E.)
   Wrote his mathematical manual. Includes "Chinese remainder problem" or "problem of the Master Sun": find *n* so that upon division by 3 you get a remainder of 2, upon division by 5 you get a remainder of 3, and upon division by 7 you get a remainder of 2. His solution: Take 140, 63, 30, add to get 233, subtract 210 to get 23.

5. Liu Hui (c. 263 C.E.)
   - Commentary on the *Nine Chapters*
     Approximates *pi* by approximating circles polygons, doubling the number of sides to get better approximations. From 96 and 192 sided polygons, he approximates *pi* as 3.141014 and suggested 3.14 as a practical approx.
     States principle of exhaustion for circles
     Suggests Calvalieri's principle to find accurate volume of cylinder
   - *Haidao suanjing (Sea Island Mathematical Manual)*. Originally appendix to commentary on Ch 9 of the *Nine Chapters*. Includes nine surveying problems involving indirect observations.

6. Zhang Qiujian (c. 450?)
   Wrote his mathematical manual. Includes formula for summing an arithmetic sequence. Also an undetermined system of two linear equations in three unknowns, the "hundred fowls problem"

7. Zu Chongzhi (429-500) Astronomer, mathematician, engineer.
   - Collected together earlier astronomical writings. Made own astronomical observations. Recommended new calendar.
   - Determined *pi* to 7 digits: 3.1415926. Recommended use 355/113 for close approx. and 22/7 for rough approx.
   - With father carried out Liu Hui's suggestion for volume of sphere to get accurate formula for volume of a sphere.



8. Liu Zhuo (544-610) Astronomer
   Introduced quadratic interpolation (second order difference method).

9. Wang Xiaotong (fl. 625) Mathematician and astronomer.
   Wrote *Xugu suanjing (Continuation of Ancient Mathematics)* of 22 problems. Solved cubic equations by generalization of algorithm for cube root.

10. Translations of Indian mathematical works.
    By 600 C.E., 3 works, since lost. Levensita, Indian astronomer working at State Observatory, translated two more texts, one of which described angle measurement (360 degrees) and a table of sines for angles from 0 to 90 degrees in 24 steps (3 3/4 degree) increments.
    Hindu decimal numerals also introduced, but not adopted.

11. Yi Xing (683-727) tangent table.

12. Jia Xian (c. 1050)
    Written work lost. Streamlined extraction of square and cube roots, extended method to higher-degree roots using binomial coefficients.

13. Qin Jiushao (c. 1202 - c. 1261)
    Shiushu jiuzhang (Mathemtaical Treatise in Nine Sections), 81 problems of applied math similar to the *Nine Chapters*. Solution of some higher-degree (up to 10th) equations. Systematic treatment of indeterminate simultaneous linear congruences (Chinese remainder theorem). Euclidean algorithm for GCD.

14. Li Chih (a.k.a. Li Yeh) (1192-1279)
    Ceyuan haijing (Sea Mirror of Circle Measurements), 12 chapters, 170 problems on right triangles and circles inscribed within or circumscribed about them. *Yigu yanduan (New Steps in Computation)*, geometric problems solved by algebra.

15. Yang Hui (fl. c. 1261-1275)
    Wrote sevral books. Explains Jiu Xian's methods for solving higher-degree root extractions. Magic squares of order up through 10.

16. Guo Shoujing (1231-1316).
    Shou shi li (Works and Days Calendar). Higher-order differences (i.e., higher-order interpolation).

17. Zhu Shijie (fl. 1280-1303)
    Suan xue qi meng (Introduction to Mathematical Studies), and *Siyuan yujian (Precious Mirror of the Four Elements)*. Solves some higher degree polynomial equations in several unknowns. Sums some finite series including (1) the sum of n^2 and (2) the sum of $n(n+1)(n+2)/6$. Discusses binomial coefficients. Uses zero digit.

Rest of outline yet to write.

Chronology of Mathematicians and Mathematical Works

Early traditional texts

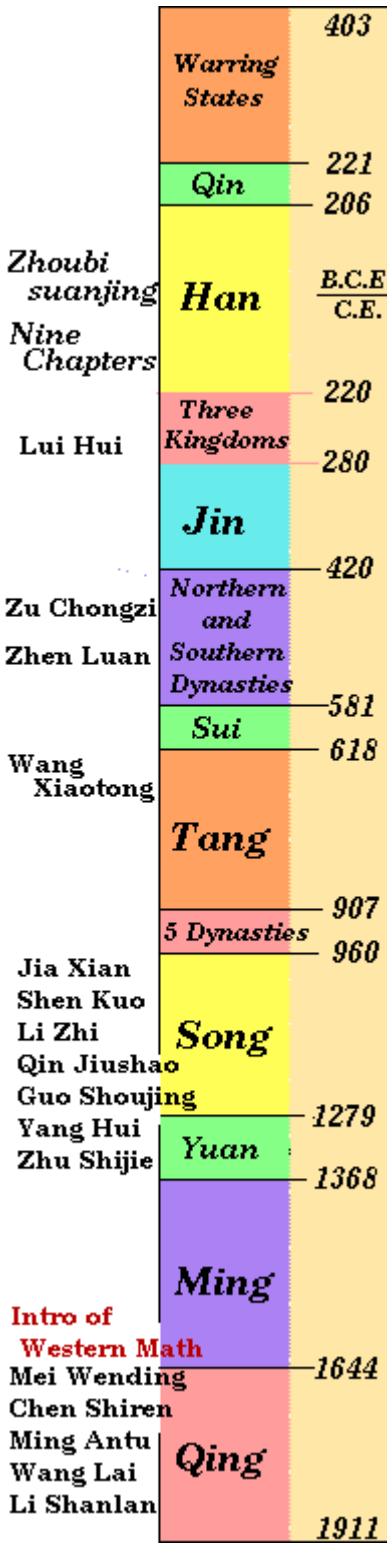
These developed in a gradual accumulation of material over centuries. The dates given are roughly when they reached their final form.

- *Suan shu shu (A Book on Arithmetic)* (c. 180 B.C.E.). A book of bamboo strips found in 1984 near Jiangling in Hubei province.
- *Zhoubi suanjing (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven)* (c. 100 B.C.E.-c. 100 C.E.)
- *Jiuzhang suanshu (Nine Chapters of the Mathematical Art)* (c. 100 B.C.E.-50 C.E.)

The development of mathematics

- Zhang Heng (78-139)
  - *Ling xian (Spiritual Constitution of the Universe)*
- Liu Hong (fl. 178-187)
  - *Qian xiang li (Calendrical Science Based on the Celestial Appearances)* (178-187)
- Wang Fan (217-257)
- Sun Zi (c. 250?)

- *Sunzi suanjing (Master Sun's Mathematical Manual)*
- Zhao Shuang (Jun Qing) (c. 260)
  - *Zhoubi suanjing zhu (Commentary on the `Zhoubi Suanjing')*
- Liu Hui (c. 263)
  - *Jiushang suanshu zhu (Commentary on the `Nine Chapters of the Mathematical Art')*
  - *Haidao suanjing (Sea Island Mathematical Manual)*
- Xiahou Yang (c. 350?)
  - *Xiahou Yang suanjing (Xiahou Yang's Mathematical Manual)*
- Zhang Qiujian (c. 450?)
  - *Zhang Qiujian suanjing (Zhang Qiujian's Mathematical Manual)*
- Zu Chongzhi (Wenyuan) (429-500)
  - *Da ming li (Da Ming Calendar)* (462)
  - *Zhui shu (Method of Interpolation)*
  - *Jiuzhang shu yi zhu (Commentary on the Methods and Essence of the `Nine Chapters')*
  - *Chong cha zhu (Commentary on Double Differences)*
- Zu Geng
  - *Zhui shu (Method of Interpolation)*
- Zhen Luan (Shuzun) (fl. 566)
  - *Tian he li (Tian He Calendar)* (462)
  - *Wucao suanjing (Mathematical Manual of the Five Government Departments)*
  - *Wujing suanshu (Arithmetic in Five Classics)*
  - *Shushu juji (Memoir on some Traditions of Mathematical Art)*
- Liu Zhuo (544-610)
  - *Huang ji li (Imperial Standard Calendar)* (600)
- Wang Xiaotong (fl. 625)
  - *Xugu suanjing (Continuation of Ancient Mathematics)*
- Li Chunfeng (fl. 664)
  - Edited the *Shibu suanjing (Ten Books of Mathematical Classics)* . This collection included the *Jiuzhang suanshu (Nine Chapters of the Mathematical Art)*, *Haidao suanjing (Sea Island Mathematical Manual)*, *Sunzi suanjing (Master Sun's Mathematical Manual)*, *Wucao suanjing (Mathematical Manual of the Five Government Departments)*, *Wujing suanshu (Arithmetic in Five Classics)*, *Zhang Qiujian suanjing (Zhang Qiujian's Mathematical Manual)*, *Xiahou Yang suanjing (Xiahou Yang's Mathematical Manual)*, *Zhui shu (Method of Interpolation)*, and *Xugu suanjing (Continuation of Ancient Mathematics)*.
- Yi Xing (683-727)
  - *Da yan li (Da Yan Calendar)* (727)
- Levensita (fl. 718)
  - *Jiu zhi li (Catching Nines Calendar)* (718) translated from an Indian work



Three century interlude

followed by

The zenith of mathematical development

- Jia Xian (c. 1050)
  - *Jia Xian suanjing (Jia Xian's Mathematical Manual)*

- Shen Kuo (1031-1095)
  - *Meng qu bi tan (Dream Pool Essays)*
- Li Zhi (Li Ye) (Jingzhai) (1192-1279)
  - *Ceyuan haijing (Sea Mirror of Circle Measurements)* (1248)
  - *Yigu yanduan (New Steps in Computation)* (1259)
- Liu Yi (fl. c. 1225)
  - *Yigu genyuan (Discussion of the Old Sources)*
- Qin Jiushao (Daogu) (c. 1202-c. 1261)
  - *Shiushu jiuzhang (Mathemtaical Treatise in Nine Sections)* (1247)
- Guo Shoujing (1231-1316)
- Yang Hui (Qianguang) (fl. 1261-1275)
  - *Xiangjie jiuzhang suanfa (A Detailed Analysis of the Mathematical Methods in the `Nine Chapters')* (1261)
  - *Riyong suan fa (Computing Methods for Daily Use)* (1262)
  - *Yang Hui suan fa (Yang Hui's Methods of Computation)* (1274)
  - *Fasuan qu yong ben mo (Alpha and Omega of a Selection on the Applications of Arithmetical Methods* (1274)
  - *Xugu zhaiqi suan fa (Continuation of Ancient Mathematical Methods for Elucidating the Strange [Properties of Numbers])* (1275)
  - *Jiuzhang suan fa zuan lei (Reclassification of the Mathematical Methods in the `Nine Chapters')*
  - *Tian mu bi lei cheng chu jie fa (Practical Rules of Arithmetic for Surveying)* (1275)
  - *Cheng chu tong bian suan bao (Precious Reckoner for Variations of Multiplication and Division)*
- Wang Xun (1235-1281)
- *Shou shi li (Works and Days Calendar)*, Guo Shoujing, Wang Xun, and others. (1280).
- Zhu Shijie (Hanqing, Songting) (fl. 1280-1303)
  - *Suan xue qi meng (Introduction to Mathematical Studies)* (1299) There is a Japanese edition of 1658.
  - *Siyuan yujian (Precious Mirror of the Four Elements)* (1303)

The decline of mathematics

- Sha keshi (fl. 1321)
  - *He fang tong yi (On the Prevention of River Flooding)* (1321)
- Ding Ju (fl. 1355)
  - *Ding ju suan fa (Ding Ju's Arithmetical Methods)* (1355)
- He Pingzi (fl. 1373)
  - *Xiangming suan fa (Explanations of Arithmetic)* (1373)
- Liu Shilong (fl. 1424)
  - *Jiu zhang tong ming suanfa (Methods of Calculation in the `Nine Chapters')* (1424)
- Wu Jing (fl. 1450)
  - *Jiu zhang suan fa bi lei da quan (Complete Description of the `Nine Chapters' on Arithmetical Techniques)* (1450)
- Wang Wensu (fl. 1524)
  - *Suan xue baojian (Precious Mirror of Mathematics)* (1524)
- Xu Xinlu
  - *Pan zhu suan fa (Method of Calculating on an Abacus)* (1573)
- Ke Sangquin (fl. 1578)
  - *Shu xue tong gui (Rules of mathematics)* (1578)
- Matteo Ricci (1552-1610)
- Niccolo Longobardi (1559-1654)
- Xu Guangqi (Zixian, Xuanhu) (1562-1633)
- Li Zhizao (Zhenzhi) (1565-1630)

Introduction of Western Mathematics

- Cheng Dawei (Rusi, Binqu)(fl. 1592)
  - *Suan fa tong zong (Systematic Treatise on Arithmetic)* (1592) Reprinted in Japan in 1675.
  - *Zhi zhi uan fa tong zong (Postscript to the Systematic Treatise on Arithmetic)* (1592)
  - *Suan fa zuan yao (Highlights of Calculation Methods* (1598)
- Huang Longyi (fl. 1604)
  - *Suan fa ji nan (Directory of Calculation Methods)* (1604)
- Johann Terrenz Schreck (1576-1630)
  - *Da ce (Complete Surveying)*
  - *Ge tu ba xian biao (Tables of Trigonometric Functions)* (1631)
  - *Ce tian yue shuo (Brief Description of the Measurement of the Heavens)*
- Li Tianjing (1579-1659)
- Translation of euclid.html">Euclid's *Elements*, first six books, Matteo Ricci and Xu Guangqi (1607)
- Giulio Aleni (1582-1649)
  - *Ji he yao fa (Essentials of Geometry)*
- Johann Adam Schall von Bell (1591-1666)
- Giacomo Rho (1593-1638)
  - *Chou suan (Napier's Bones)* (1628)
  - *Ce liang quan yi (Complete Theory of Surveying)* (1631)
  - *Bi li gui jie (Manual for proportional dividers)* (1631)
- *Tong wen suan zhi (Treatise on European Arithmetic)* an edited translation of Clavius's *Epitome of Practical Arithmetic*, Matteo Ricci and Li Zhizao (1631)
- *Chong zhen li shu (Chong Zhen Reign Treatise on Astronomy and Calendrical Science).* (1631-1634).
  A collection of 137 books in five submissions edited by Xu Guanqi and Li Tianjing with support of many others. It included Aleni's *Ji he yao fa (Essentials of Geometry)*; Terrenz's *Da ce (Complete Surveying)*, *Ge tu ba xian biao (Tables of Trigonometric Functions)*, and *Ce tian yue shuo (Brief Description of the Measurement of the Heavens)*; and Rho's *Ce liang quan yi (Complete Theory of Surveying)*, *Bi li gui jie (Manual for proportional dividers)*, and *Chou suan (Napier's Bones)*.
- Jean Nicolas Smogulecki (1611-1656)
- *Xi yang xin fa li shu (Treatise on Astronomy and Calendrical Science According to the New Western Methods).*
  A collection of 100 books in 17 volumes emended by Schall von Bell from the *Chong zhen li shu (Chong Zhen Reign Treatise on Astronomy and Calendrical Science)* (1645)
- Xue Fengzuo (d. 1680)
- Fang Zhongtong (1633-1698)
- *Li xue hui tong (Understanding Calendar Making)* (1652-1654)
  A collection of books published by Smogulecki and Xue Fengzuo. Included are
  - Bi li si xian xin biao (New Tables for Four Logarithmic Trigonometric Functions)
  - Bi li dui shu biao (Logarithm Tables with Explanations)
  - San jiaofa (Essentials of Trigonometry) (1653)
- *Tian bu zhen yuan (True Course of Celestial Motions)* (1653)
  A collection of books written by Smogulecki and Xue Fengzuo. Includes
  - *San jiao suan fa (Method of Trigonometrical Calculations)*
- Mei Wending (Dingjiu, Wu'an) (1633-1721)
  - See Mei Juecheng, *Mei shi congshu jiyao (Collected Works of the Mei Family)* for publication of Mei Wending's written comments on mathematics
- *Shu li jing yun (Collected Basic Principles of Mathematics)* (1723).
  Supervised by Emperor Kang Xi (Aixinjueluo) (1654-1722), edited by Mei Juecheng, Chen Houyao, He Guozong, Ming Antu, Mei Wending, and others.
- Mei Juecheng
  - In 1761, Mei Juecheng complied Mei Wending's written commentaries into the *Mei shi congshu jiyao (Collected Works of the Mei Family)*. It included several works on mathematics: *Bisuan (Pen Calculations)*, *Chou suan (Napier's bones)*, *Du suan shi li (Proportional Dividers)*, *Shao guang shi*

yi (Supplement to `What Width'), *Fang cheng lun (Theory of Rectangular Arrays)*, *Gougu ju yu (Right-angled Triangles)*, *Jihe tong jie (Explanations in Geometry)*, *Ping san jiao ju yao (Elements of Plane Trigonometry)*, *Fang yuan mi ji (Squares and Circles, Cubes and Spheres)*, *Jihe bu bian (Supplement to Geometry)*, *Hu san jiao ju yao (Elements of Spherical Trigonometry)*, *Huan zhong shu chi (Geodesy)*, and *Qiandu celiang (Surveying Solids)*.

Mathematics under the "Closed Door" Policy

- Chen Shiren (1676-1722)
  - *Shao guang bu yi (Supplement to `What Width')*
- Ming Antu (d. 1765)
  - *Suanjing shishu (Ten Mathematical Manuals)* (1773)
  - *Ge yuan mi lu jie fa (Quick Method for Determining Close Ratios in Circle Division)* (1774)
- Jiao Xun (1763-1820)
  - *Da yan qiu yi shu (Technique for Finding 1 by the Great Extension)*
- Ruan Yuan (1764-1849)
- Wang Lai (Xiaoying, Hengzhai) (1768-1813)
  - *Hengzhai suanxue (Hengzhai's Mathematics)*
  - *Hengahai yi shu (Unpublished Works of Hengzhai)* (1834, edited by Xia Xie)
- *Chou ren zhuan (Biographies of Mathematicians and Astronomers)* (1795-1799). Edited by Ruan Yuan.
- Li Huang (d. 1811)
  - *Jiuzhang suanshu xi cao tu shuo (Careful Explanation of the `Nine Chapters on the Mathematical Art')*
  - *Haidao suanjing xi cao tu shuo (Careful Explanation of the `Sea Island Mathematical Manual')*
  - *Xu gu suanjing kao zhu (Commentary on the `Continuation of Ancient Mathematical Methods for Elucidating the Strange [Properties of Numbers]')*
- Li Rui (Shangzhi, Sixiang) (1773-1817)
  - *Li shi suan xue yi shu (Collected Mathematical Works of Li Rui)*
- Luo Tengfeng
  - *Yi you lu (Records of the Art of Learning)* (1815)
- Xiang Mingda (1789-1850)
  - *Xiang shu yi yuan (The Source of Series)* (1888, edited by Dai Xu)
- Luo Shilin (1789-1853)
  - *Siyuan yujian xicao (Commentary on the `Precious Mirror of the Four Elements')* (1836)
- Dong Youcheng (Fangli) (1791-1823)
  - *Ge yuan mi lu tu jie (Explanation for the `Determination of Close Ratios in Circle Division')*
- Gu Guanjuang (1799-1862)
  - *Zhoubi suanjing xiao kan ji (A Textual Criticism of the `Zhoubi Suanjing')*
- Shen Qinpei (fl. 1829)
  - *Siyuan yujian xicao (Commentary on the `Precious Mirror of the Four Elements')* (1829)
- Zhang Dunren (fl. 1831)
  - *Qiu yi suan shu (Techniques of Finding 1)* (1831)
- Dai Xu (1805-1860)
  - *Dai shu jian fa (Concise Technique of Logarithms)* (1846)

International Mathematics in China

- Li Shanlan (Renshu, Qiuren) (1811-1882)
  - *Duo ji bi lei (Sums of Piles of Various Types)*
  - *Fang yuan chan you (Explanation of the Square and the Circle)*
  - *Hu shi qi mi (Unveiling the Secrets of Arc and Sagitta)*
  - *Dui shu tan yuan (Seeking the Source of Logarithms)*
  - Several translations of Western mathematics (1852-1866)

- Hua Hengfan (Ruo Ting) (1833-1902)
  - *Xingsu xuan suan (Mathematical Papers form the Xing Su Study)*
  - Several translations of Western mathematics (1868-1886)
- Shi Richun
  - *Qiu yi shu zhi (Path to the Technique of Finding 1)* (1873)
- Huang Zongxian
  - *Qiu yi shu tong jie (Explanation of the Technique of Finding 1)* (1873)

Some Selected On-line References

- Erik Peterson`s Chinese Numbers page
- History of China page, part of the China page. (Robert Felsing, University of Oregon Library)
- Heng Yuan's Chinese Web Directory
- Patrick Kremer's China related WWW resources catalogue ZWWWZ

Bibliography

- Dereke Bodde. *Chinese Thought, Society and Science: The Intellectual and Social Background of Science and Technology in Pre-Modern China*. University of Hawaii Press, Honolulu, 1991.
  Reviewed by Shigehisa Kuriyama in *Configurations* 1 (1993). (The journal *Configurations* is available on-line at Project Muse of the Johns Hopkins University Press.)
- Yan Li and Shi Ran Du. *Chinese Mathematics, a Concise History,* Translated from the Chinese by John N. Crossley and Anthony W.-C. Lun. Clarendon Press, Oxford, 1987. Review: *Math. Rev.* 90m:01003.
- Ulrich Libbrecht. *Chinese mathematics in the thirteenth century.* MIT Press, Cambridge, Mass, 1973.
- Yoshio Mikami. *The Development of Mathematics in China and Japan,* 2nd edition, Chelsea Publ., New York, 1974. (1st ed. Leipzig, 1913.)
- Shigeru Nakayama. *Chinese science; explorations of an ancient tradition.* Edited by Shigeru Nakayama and Nathan Sivin. MIT Press, Cambridge, Mass., 1973
- Joseph Needham with the collaboration of Wang Ling. *Science and civilization in China.* Vol. 3: *Mathematics and the Sciences.* Cambridge Univ Press, New York, 1959.
- Nathan Sivin.
  - *Cosmos and early Chinese mathematical astronomy.* Brill, Leiden, 1969.
  - *Science and technology in East Asia.* Edited by Nathan Sivin. Science History Publications, New York, 1977.
- Frank J. Swetz. *The Sea Island Mathematical Manual: surveying and mathematics in ancient China.* Pennsylvania State Univ. Press, University Park, Pa., 1992.
- Frank J. Swetz and T. I. Kao. *Was Pythagoras Chinese?* National Council of Teachers of Mathematics, Reston, Virginia, 1977. Reprint: Pennsylvania State Univ. Press, University Park, PA, 1980.

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Initial work December, 1994. Latest update Sept 17, 1995.