

# Outline of the History of Mathematics in India

Math 105 History of Mathematics

Prof. D. Joyce, Clark University, Nov 2006

## Sources.

C. N. Srinivasiengar. *The History of Ancient Indian Mathematics*. World Press Private Ltd., Calcutta, 1967.

Sarasvati. *Geometry in Ancient and Medieval India*. Indological Publ., Delhi, 1979.

## 1. Numerals.

(a) Early numeration was base 10, non-place value

(b) Base 10 place value notation

595 C.E. a place value notation used: 346

875 C.E. complete place value notation with zero digit

ca. 750 C.E. a zero digit used

ca. 750 C.E. Chinese reference to Indian place value notation with 0 digit

683 C.E. zero digit used in Hindu colonies in Khmer (Cambodia) and Sumatra

Later full system adopted by Arabs and 0 digit adopted by Chinese

## 2. Mathematics in the Sulvasutras. Hindu spiritual texts ca. 800–200 B.C.E.

Description of altar and fireplace construction describes areas of rectangles, parallelograms, isosceles trapezoids. States and uses Pythagorean theorem (no proof), similar triangles. Some Pythagorean triples 3:4:5, 5:12:13, 7:24:25, 8:15:17, 12:35:37. Specific constructions (1) perpendicular bisectors and constructing a perpendicular to a point, (2) constructing squares and trapezoids, (3) interconverting squares and circles (with  $\pi$  as 3.088 or 3.004), (4) constructing a square  $n$  times the area of a given square, (5) constructing a square the sum or difference of two given squares, (6) interconverting squares, rectangles, trapezoids, and isosceles triangles. Common fractions. Some simultaneous linear indeterminate equations.

## 3. Mathematics of the Jainas. (religion)

Gantanuyoga, system of calculation, ca. 500–300 B.C.E.  $\pi$  approximations include 3 and  $\sqrt{10}$ . Umaswati (fl. ca. 150 B.C.E.) wrote texts and started math. school at Pataliputra. Circle =  $\frac{1}{4}$  circumference times diameter. Area of circle segments. Isosceles trapezoids. Quadratic equations. Arithmetic of infinity (indefinitely large number). Square and Cube roots. Combinatorial problems involving combinations and permutations.

4. **Siddhantas.** Hindu systems of astronomy, in particular, *Surya Siddhantaca* (*System of the Sun*). 400 C.E.  $\pi$  approximations include  $\sqrt{10}$  and  $3\frac{177}{1250}$ . Tables of sines for 24 equal parts of a right angle (every  $3\frac{3}{4}$  degrees), perhaps derived from Hipparchus' tables.
5. **Aryabhata**, (b. 476 C.E) *Arybhatiya*, 499 C.E. At school at Pataliputra. Square and cube root algorithms. Some geometrical problems. Quadratic equations. Described general integer solutions to linear indeterminate equations using a variant of the Euclidean algorithm. approx. For  $\pi$ , 3.1416 preferred to root 10. Table of sines. Used a kind of alphabetic numerals where consonants play digits and vowels play powers of 100. Summed squares, cubes, and triangular numbers. Incorrect volume of sphere ( $\pi\sqrt{\pi}r^3$ ), and triangular pyramid ( $\frac{1}{2}$  height x Base)
6. **Brahmagupta**, 598  
West India (Sind). *Brahma Sphuta Siddhanta* long work on astronomy with  $4\frac{1}{2}$  chapters on math based on an older astronomical work *Brahma Siddhanta*. Translated by Arabs in 770 as *Sind Hind*. Fully accepted zero and negative numbers. Solved what's now called the Pell equation, the indeterminate equation  $Nx^2 \pm 1 = y^2$ . Summed finite geometric progression. General solution to  $x^2 + y^2 = z^2$ , i.e., found all Pythagorean triples. Geometric constructions including circle circumscribing and inscribed in triangles. Cyclic quadrilaterals: constructing those with rational sides, Area =  $\sqrt{(s-a)(s-b)(s-c)(s-d)}$  where  $s$  is semiperimeter, problems with isosceles trapezoids. *Khandakhadyaka*, 665. Expository astronomy describes construction table of sines using second order differences (as did Aryabhata). Syncopated algebra.
7. **Mahaviracharya.** Jain mathematician. *Ganita Sara Sangraha*, 850  
Standard problems. Some 4th degree equations. General formula for combinations  $\binom{n}{k}$  and permutations of  $n$  things  $k$  at a time. Problems with unit fractions. Indeterminate equations. Cyclic quadrilaterals. Mentions ellipse, but gives wrong area.
8. **Halayudha.** Jain mathematician 10th century d Describes arithmetic triangle (Pascals triangle).
9. **Bhaskara**, (1114–ca. 1185) *Siddhanta Siromani*, 1150  
Four parts: *Lilavati* arithmetic, *Bijanganita* algebra, *Goladhyaya* celestial globe, *Gruha-ganita* mathematics of the planets. Arithmetic includes 0, negatives, and infinity. Operations on surds, e.g.,  $\sqrt{ab} = \sqrt{a}\sqrt{b}$ ,  $\sqrt{a} + \sqrt{b} = \sqrt{a+b+2\sqrt{ab}}$ . Recognized two roots of quadratic equations even when one negative. Correct area and volume of sphere. Combinatorics: permutations of  $r$  things,  $m$  of one type,  $n$  of another,  $\dots = \frac{r!}{m!n!\dots}$ . Problems on right triangles. Trigonometry: sine of a sum of two angles, double angle formula. Infinitesimal methods to compute area of sphere (both by dividing sphere into annuli and lunes) and volume of sphere (by dividing into pyramids with apex at center of sphere), to compute motion by dividing time into a large number of parts, and even some principles for differentials, e.g.,  $\sin y' - \sin y = (y' - y) \cos y$  when  $y'$  is nearly  $y$ . Noted when a planet's motion is an extremum, it is stationary.

This outline only goes to Bhaskara, but mathematics developed much after Bhaskara.