9.B. Ohm's Law and alternating current circuits

In the files `impedance.pdf` and `impedance2.pdf` we noted that arithmetic with complex numbers plays an important role in working with alternating current (AC) circuits because the AC counterpart of resistance — namely, *impedance* — takes values in complex numbers rather than real numbers. One illustration of the resistance/impedance analogy is that Ohm's Law for direct current (DC) computing resistances in parallel circuits

$$\frac{1}{R} \quad = \quad \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$$

has an analog in which the resistances R and R_k are replaced by impedances Z and Z_k. We shall work a typical problem involving parallel impedances here.

PROBLEM. *Find the total impedance Z if the circuit has $n = 3$ branches and $Z_k = k + i$, where $i^2 = -1$ (as is usuall the case in mathematics).*

SOLUTION. At many points we shall need the following identity from `impedance.pdf`:

$$\frac{1}{x + y\,i} \quad = \quad \frac{x - y\,i}{x^2 + y^2}$$

If we choose Z_k as above, we obtain the equation

$$\frac{1}{Z} \quad = \quad \frac{1}{1+i} + \frac{1}{2+i} + \frac{1}{3+i} \quad = \quad \frac{1-i}{2} + \frac{2-i}{5} + \frac{3-i}{10}$$

and if we simplify this we see that

$$\frac{1}{Z} \quad = \quad \frac{5(1-i)}{10} + \frac{2(2-i)}{10} + \frac{3-i}{10} \quad = \quad \frac{12-8i}{10} \;.$$

Therefore we have

$$Z \quad = \quad \frac{10}{12-8i} \quad = \quad \frac{10 \cdot (12+8i)}{12^2 + 8^2 = 208} \quad = \quad \frac{3-2i}{54} \;.$$