Coordinate geometry and Aristotle's Theorem

One example showing the power of coordinate methods in geometry is the following result which appears in the writings of Aristotle:

THEOREM. *Let A and B be two points in the plane, and let r be a real number such that $r > 1$. Then the locus (= set) of all points X in the plane such that the distances from X to A and B satisfy*

$$\frac{|XA|}{|XB|} = r$$

is a circle,

This was previously assigned as an exercise, and a proof was given in terms of coordinate geometry. The first step is to choose a coordinate system such that the line AB is the x-axis and A is the origin, and with this choice the algebra of the problem immediately yields information like the location of the circle's center and the circle's radius. In contrast, if one tries to use the synthetic (classical Greek) methods, then one must begin by finding the center and radius of the circle, after which one can apply suitable propositions from Euclid. Of course, this step is also nontrivial. This strongly suggests that a synthetic proof would be far more difficult than an analytic proof using coordinates, and in fact this is true. A more complicated example is worked out in Burton, and there are four files in the folder

`http://math.ucr.edu/~res/math153-2020/week9/unit11`

which work numerous locus problems at various levels of difficulty.

The Internet site

`https://plato.stanford.edu/entries/aristotle-mathematics/supplement4.html`

contains an animated figure for the theorem stated above, and it also contains a considerable amount of additional information about the mathematics which appears in Aristotle's writings.