Comments on `locus-problems.pdf`

In the cited document, we gave two proofs that the locus of points which are equidistant from a pair of parallel lines is a third line, which is parallel to the given two lines and halfway between them. We also noted that the proof using coordinate geometry was much shorter and direct than the proof using classical methods from Greek geometry. In fact, the difference in required amounts of work may suggest that we are getting something for nothing by using coordinate geometry. However, as in other contexts, it is very rare in mathematical reasoning to get something for nothing, and if this appears to be the case it is definitely worth examining the situation further to understand why this SEEMS to be happening.

In the classical proof of the locus theorem, at several points we needed the following three results:

> *The opposite sides of a rectangle have equal lengths.*
>
> *If two coplanar lines are each perpendicular to a third line (in the same plane), then the original two lines are parallel.*
>
> *If we are given two parallel lines and a third line in the same plane, then the latter line is perpendicular to one of the two parallel lines if and only if it is perpendicular to the other.*

All of these statements are implicit in the setup of coordinate systems. We could not choose coordinate systems with the usual algebraic interpretation of distances if they were not true. Therefore, the statement, "choose coordinate axes such that ... ," strongly depends upon the validity of these three results even though it might not be apparent from the reasoning in the argument. The three statements do not appear explicitly because they lie beneath the surface as steps in the proof that good coordinate systems actually exist.