USING LOGARITHMS

The examples below are taken from pp. 404 - 405 of

M. P. Dolciani, W. Wooton, E. F. Beckenbach and S. Sharron, Algebra 2 and Trigonometry (Rev. Ed.). Houghton Mifflin, Boston, 1983.

For centuries logarithms were used extensively to simplify numerical computations, converting multiplication problems into addition problems. The power and availability of electronic computing devices has made this usage obsolete in most situations, but for historical purposes it is enlightening to see examples of such computations.

Example 1 Compute $\dfrac{438 \times 0.410}{2.32}$

using (common) logarithms.

Method We are given A, B, C and want $\dfrac{AB}{C}$. Start with

$$A = 10^{\log_{10} A}, \quad B = 10^{\log_{10} B}, \quad C = 10^{\log_{10} C}.$$

Then $\frac{AB}{C} = \frac{10^{\log_{10} A} \cdot 10^{\log_{10} B}}{10^{\log_{10} C}} =$

$\frac{10^{\log_{10} A + \log_{10} B}}{10^{\log_{10} C}} = 10^{\log_{10} A + \log_{10} B - \log_{10} C}$

So for given A, B, C we

1. look up $\log_{10} A$, $\log_{10} B$, $\log_{10} C$ in a table.
2. Compute $X = \log_{10} A + \log_{10} B - \log_{10} C$.
3. Find $Y = \text{antilog}_{10} X$ such that $\log_{10} Y = X$, or equivalently $Y = 10^X$.
4. Observe that Y in 3 equals $\frac{AB}{C}$.

<u>Application to Example 1</u>

If $A = 438$, then $\log_{10} A \approx 2.6415$.

If $B = 0.410 = 4.1 \times 10^{-1}$, $\log_{10} B \approx 0.6128 - 1$

(Note how we have to re write B)

If $C = 2.32$, then $\log_{10} C \approx 0.3655$

This means $X \approx 1.8888$, and hence

$$Y = \text{antilog}_{10} X = \text{antilog}_{10} 1.8888 \approx \boxed{77.42}$$

Example 2 Compute $\frac{7.08}{-15.9}$ similarly.

Logarithms only work when all terms are positive, so we have to compute $\frac{7.08}{15.9}$ and take its negative.

IMPORTANT. When denominator > numerator, take a power of 10, say 10^k, such that $10^k N > D$. Then compute $10^k N/D$ as before, and obtain N/D by dividing by 10^k.

In Example 2, $10 \cdot N > D$, so take $k = 1$.

Then $\log_{10} 70.8 = 1.8500$

$\log_{10} 15.9 = 1.2014.$

Then $\dfrac{70.8}{15.9} = 10^{\log_{10} 70.8 - \log_{10} 15.9} = Y$

Compute $X = \log_{10} 70.8 - \log_{10} 15.9 \approx 0.6486$.

Then $Y = 10^X$, or $Y = \operatorname{antilog}_{10} X \Rightarrow$

$$Y \approx 4.45$$

Hence $\dfrac{7.08}{15.9} = 10^{-1} Y \approx 0.445$

and $\dfrac{7.08}{15.9} \approx -0.445$.