VERNIER SCALES

The basic idea involves two scales of measurement. One is a normal scale subdivided ~~into~~ with ~~marks~~ rulings every $1/N$ units for some positive integer N. The other is ~~a~~ a piece that is $N - 1/N$ units long with ~~rulings~~ $N+1$ equally spaced rulings (hence adjacent rulings are $(N-1)/N^2$ units apart).

To measure an object, one begins by using the first ruling to determine the positive integer x such that the measurement m is between $\frac{x}{N}$ and $\frac{x+1}{N}$. Assume that $m = \frac{y}{N^2}$ for some y. One then slides the second scale so that the left hand end is at $m = \frac{y}{N^2}$ on the scale. Then there will be some integer k such that $0 \leq k < N$ and the k-th ruling will coincide with some point z/N.

<u>CLAIM:</u> $m = \dfrac{Nx + k}{N^2}$.

VERIFICATION. Write $y = aN + b$ where $a \geq 0$ and $0 \leq b < N$. Then we must have $a = x$.

The distance from the 0^{th} to k^{th} ruling is equal to $\frac{k \cdot (N-1)}{N^2}$, so we are saying that $\frac{b}{N^2} + \frac{k(N-1)}{N^2}$ is a fraction with denominator N. In other words

$b + kN - k$ is divisible by N, or equivalently

$b - k$ is divisible by N. Since $0 \le b, k < N$ we know that $-N < b - k < N$, so that $b - k$ must be zero and hence

$$\frac{y}{N^2} = \frac{aN}{N^2} + \frac{b}{N^2} = \frac{x}{N} + \frac{k}{N^2}.$$

Standard examples:

Metric The units are centimeters with $N = 10$. Then one gets measurements accurate to 0.01 cm $=$ 0.1 mm.

"American" The units are half-inches with $N=8$ (standard $1/16$ inch rulings). Then one gets measurements accurate to $1/64$ half-inches or $1/128$ inches.