

MORE EXERCISES RELATED TO history06X.pdf

Additional exercises

21. For each of the following numbers n , find a one or two digit integer c such that $101c \equiv 1 \pmod{n}$. You may use the Euclidean Algorithm or a calculator to solve this problem.

- (a) $n = 97$
- (b) $n = 99$
- (c) $n = 103$
- (d) $n = 105$

22. (a) Use the methods of **history06b.pdf** to show that if p is a prime and $x = a + b\sqrt{p}$ is a number such that a and b are rational numbers with $b \neq 0$, then $x \neq 0$ and

$$\frac{1}{a + b\sqrt{p}} = \frac{a - b\sqrt{p}}{N(a + b\sqrt{p})}.$$

(b) Suppose that integers u and v satisfy the equation $u^2 = v^2p - 1$, where once again p is a prime. Construct a solution to the Pell's equation $a^2 = b^2p + 1$.