Partial solutions for Quiz 2

1. We want to find a reasonably small value of c such that $101c \equiv 1 \bmod 107$; more precisely we would like to find some choice of c which is a one or two digit positive integer. We can do this as follows:

$101 \equiv -6 \bmod 107$

$108 = 107 + 1 = 6 \times 18$

$101 \cdot 18 \equiv\equiv -6 \cdot 18 \equiv -1 \bmod 107$

Therefore $c \equiv -18 \bmod 107$

But $89 = 101 - 18 \equiv -18 \bmod 107$

Therefore $c \equiv 89 \bmod 107$■

2. We need to solve the simultaneous congruences $x \equiv P \bmod 101$ and $x \equiv Q \bmod 107$. We can write $x = P + 101n$, where we wish to describe n more precisely. We also have that $x \equiv Q \bmod 107$, and this yields a second relationship

$$x \quad = \quad P \ + \ 101n \quad \equiv \quad Q \bmod 107$$

and we can now use the first problem to conclude that $89(Q - P) \equiv 89 \cdot 101n \equiv n \bmod 107$. This means that $n \equiv 89(Q - P) \bmod 107$. This yields one form of the final answer:

$$x \quad \equiv \quad P \ + \ (101 \cdot 89) \cdot (Q - P) \ + \ 10807m$$

Here m is an arbitrary integer. We can now replace $P \ + \ (101 \cdot 89) \cdot (Q - P)$ by an integer R which is congruent to it mod 10807 and lies between 0 and 10806, so that $x \equiv R \bmod 10807$.■