NAME: ______________________________

Mathematics 153, Spring 2021, Examination 2

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail, one to **EACH of the following THREE** addresses, by **11:59 P.M.** on **Wedenesday, June 9, 2021:**

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Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. Outside references such as course directory documents may be used, and you may discuss the problems informally with other students or ask for clarifications from either the teaching assistants or me, but the writeup you submit is required to be your own work.

The nominal top score for setting the curve will be 150 points.

1. [*25 points*] Given a positive integer x, let $f(x)$ denote the following partial fraction expression:

$$1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{x}}}$$

Show that if $x < y$ then $f(x) < f(y)$. [*Hints:* Simplify the right hand side into a quotient of two polynomials, and recall that if A, B, C, D are positive real numbers, then $A < B < C/D$ if and only if $AD < BC$.]

2. [*25 points*] (a) Let p be a prime. Prove that if the integers a, b satisfy $a \equiv b \mod p$ then $a^p \equiv b^p \mod p^2$. [*Hint:* Look at the proof of the Little Fermat Theorem.]

(b) Let Give an example of an even positive integer n and an integer a such that $a^n \not\equiv a \mod n$. By the Little Fermat Theorem we must have $n > 2$.

3. [*25 points*] (a) The function $y(x) = 2x^3 + 3x$ is a strictly increasing function from the nonnegative real numbers to itself which is 1–1 onto, and hence it has an inverse function. Use the Cubic Formula to give an explicit formula the inverse function $x(y)$ involving the four basic arithmetic operations and taking square and cube roots.

(b) It is known that the integral function

$$\int_0^t \frac{dt}{\sqrt{1-t^4}}$$

cannot be expressed in terms of the functions which arise in first year calculus. Using Newton's Binomial Series find the first four nontrivial terms of an infinite series for this function involving powers of x (*i.e.*, a partial Maclaurin series).

4. [*25 points*] Let L_0 and L_1 be the vertical lines in the coordinae plane defined by $x = 0$ and $x = 1$ respectively. Find the locus (or set) of all points P in the coordinate plane such that the (shortest) distance from P to L_0 is the **square** of the (shortest) distance from P to L_1, and draw a sketch of the locus.

5. [*30 points*] For **TEN** of the contributors to the history of mathematics listed below, give the century in which he or she was active. Extra credit may be obtained if more than 10 are answered correctly.

Al-Hazen

Al-Kashi

Bhaskara (a.k.a. Bhaskara II)

Bombelli

Brahmagupta

Fibonacci

Heron (a.k.a. Heron)

Oresme

Pappus

Seki

Thabit ibn Qurra

Viète

6. [*20 points*] (*a*) State whether the assertion

On a sphere, a 45° *latitude circle is a great circle*

is true or false and give reasons for your ahswer.

(*b*) Name two logical issues which were either overlooked in Euclid's *Elements* or were done particularly well, and give reasons for your answer. You may take two issues of one kind or one issue of each kind, and extra credit is possible if three or four issues are properly identified.