SOLUTIONS TO EXERCISES FROM `math153exercises03a.pdf`

6. Here are tables which display the steps in applying the Euclidean Algorithm**.** The greatest common divisors are highlighted in

green.

a	b	q	r
662	414	1	248
414	248	1	166
248	166	1	82
166	82	2	**2**
82	2	41	0

a	b	q	r
277	123	2	31
123	31	3	30
31	30	1	**1**
30	1	30	0

a	b	q	r
201	111	1	90
111	90	1	21
90	21	4	6
21	6	3	**3**
6	3	2	0

a	b	q	r
5040	1001	5	35
1001	35	28	21
35	21	1	14
21	14	1	**7**
14	7	2	0

a	b	q	r
9998	6060	1	3938
6060	3938	1	2122
3938	2122	1	1816
2122	1816	1	306
1816	306	5	286
306	286	1	20
286	20	14	6
20	6	3	**2**
6	2	3	0

a	b	q	r
14039	1529	9	278
1529	278	5	**139**
278	139	2	0

a	b	q	r
54321	12345	4	4941
12345	4941	2	2463
4941	2463	2	15
2463	15	164	**3**
15	3	5	0

a	b	q	r
111111	11111	10	**1**
11111	1	11111	0

7. (*a*) We have $(2k+3) - (2k+1) = 2$ and $(2k+1) - 2*k = 1$, which yield the identity $1 = (k+1)*(2k+1) - k*(2k+3)$. Since the greatest common divisor d of m and n also divides every integral linear combination $x*m + y*n$, it follows that d divides **1** and hence d must be **1.**

(*b*) In this case we have $(2k+5) - (2k+1) = 4$, but now we must consider the cases where k is even or k is odd separately**.** Suppose first that k is even and write $k = 2r$. Then we have $1 = (2k+1) - 4*r = (r+1)*(2k+1) - r*(2k+5)$. The same reasoning as in part (a) now implies that the greatest common divisor must be **1.**

Next suppose that k is odd and write $k = 2r+1$ so that $2k+1 = 4r+3$. In this case we have $3 = (2k+1) - 4*r$ and $4 - 1*3 = 1$, which imply that $1 = (r+1)*(2k+5) - (r+2)*(2k+1)$. The same considerations as before now show that the greatest common divisor must be **1.**

8. The simplest example occurs when $k = 1$, in which case $2k+1 = 3$ and $2k+7 = 9$. In fact, the two numbers are not relatively prime if and only if $k = 3r + 1$ for some nonnegative integer r. (See if you can prove this!)