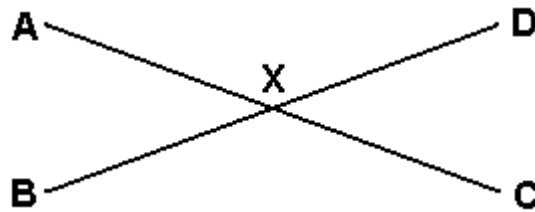


The Vertical Angle Theorem

Here is the standard proof of this result using synthetic (= classical Greek) methods. The proof of this result is attributed to Thales of Miletus. All of the geometrical notation in this discussion is described in the following file:

<http://math.ucr.edu/~res/math153-2020/week1/history00g.pdf>

Proposition. (Vertical Angle Theorem) *Let A, B, C, D be four distinct points in the plane such that the betweenness relations $A * X * C$ and $B * X * D$ are valid. Then the measures of the vertical angles satisfy $|\angle AXB| = |\angle CXD|$.*



Proof. Two applications of the Supplemental Angle Identity (the sum of their measures is 180 degrees) imply that

$$|\angle AXB| + |\angle AXD| = 180 = |\angle DXA| + |\angle DXC|$$

and if we subtract $|\angle AXD| = |\angle DXA|$ from the left and right hand side we obtain

$$|\angle AXB| = |\angle DXC|$$

which is equivalent to the stated conclusion because $\angle CXD = \angle DXC$. ■