Remark on a proof in the *Elements*

In the ***Elements***, Euclid gave a simple proof of the following result, and in many respects his proof is still the best one:

<u>THEOREM.</u> ***There are infinitely many (positive) primes.***

The proof is by contradiction. Assuming that there are only n primes $p_1, \ldots, p_n$ Euclid shows that $(p_1 \ldots p_n) + 1$ is not divisible by any of them, and since every integer greater than 1 is a product of primes there must be a prime not equal to any of the p_j's, a contradiction which shows that there must be infinitely many primes.

Unfortunately, there is a common misunderstanding of exactly what the argument implies. Namely, the argument does **<u>NOT</u>** imply the following statement:

If $p_1, \ldots, p_n$ are the first n primes, then $(p_1 \ldots p_n) + 1$ is also prime.

The simplest way to disprove this statement is to note that

$$(2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13) + 1 = 30031 = 59 \cdot 509$$

is not prime.

This and other common misconceptions are described in the following reference:

http://mathoverflow.net/questions/23478/examples-of-common-false-beliefs-in-mathematics