## Some basic figures in solid geometry

Here are some of the sets which are fundamentally important in solid geometry.


Line perpendicular to a plane. This means that the line $P Q$ is perpendicular to every line in the plane which passes through $\mathbf{Q}$; in fact, the latter is true if and only if $\mathbf{P Q}$ is perpendicular to two distinct lines which are in the plane and pass through $\mathbf{P}$.


Space separation. A plane separates 3 - space into two regions or "sides," similar to the way in which a line separates a plane. This was not always explicitly recognized in Greek geometry.


Parallel planes. In analogy with plane geometry, given a plane $\mathbf{E}$ and a point $\mathbf{X}$ not on the plane there is a unique plane $\mathbf{F}$ such that $\mathbf{X}$ lies in $\mathbf{F}$ and the planes $\mathbf{E}$ and $\mathbf{F}$ have no points in common.


Perpendicular planes. Each of the three lines in the drawing is perpendicular to the other two. This is related to the concept of dihedral angle described below.


Skew lines. These are lines which do not meet but are not coplanar (by definition, parallel lines are coplanar). Given two such lines, the shortest distance between them is the length of a (unique) common perpendicular segment with one endpoint on each line.


Dihedral and trihedral angles. These are shown on the left and right respectively, and despite the similarity of their names they are clearly different types of objects. In the left hand drawing the lines $\mathbf{A B}$ and $\mathbf{B C}$ are perpendicular to the line in which the planes intersect, and the dihedral angle's measure is given by the angle ABC. The dihedral angle is a union of two planar sets (the closed half planes), and the trihedral angle is a union of three coplanar sets (the interiors of the angles which have $\mathbf{A}$ as a vertex together with the three planar angles themselves). Solid geometry also studies more general polyhedral angles like the apex of a pyramid with a square base (see the next page).


This seems like a good place to stop. Of course, there are also numerous figures which arise in the study of the sphere. Several of them are discussed in Section V. 1 of the following online document:
https://math.ucr.edu/~res/math133-2020/week8/geometrynotes05a.f13.pdf

