COMMENTS ON CONGRUENCE AND SUPERPOSITION

The following passage from page 252 of Moïse, *Elementary Geometry from and Advanced Standpoint* (Third Edition), summarizes the viewpoint in Section II.4 of the notes. In some sense these elaborate on the citation from Schopenhauer which appears in the notes.

> Euclid based all of his congruence proofs on a statement that said that "things which coincide with each other are equal to each other" ... This was not adequate to account for the things that Euclid actually did. Strictly speaking, figures only coincide with themselves. And it is plain that the idea of motion, or superposition, is implicit in Euclid's congruence proofs. Some authors [including many high school textbook writers] have attempted to make this idea explicit by stating a postulate to the effect that "geometric figures can be moved without changing their size or shape." But this is still not enough; it clarifies the difficulty without removing it. The difficulty is that while the term *figure* is plain enough (a figure is a set of points) the terms *moved*, *size*, and *shape* have an insecure status. [In the absence of further discussion] They must be regarded as undefined terms, since no definitions have been given for them. But if they are undefined, then postulates must be given, conveying their essential properties, and this has not been done either. The general drift of the postulate [or list of postulates] is plain, but you cannot base a mathematical proof on a general drift.
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> It is possible, however, to formulate Euclid's idea in an exact mathematical way. We ... do this, by **defining** [*emphasis added*] the general idea of *rigid motion*, or *isometry*.

This notion is defined and studied in Section II.4 of the notes. The idea of an isometry (a $1-1$ correspondence which preserves distances) also appeared earlier in the statement of the Ruler Postulate at the beginning of Section II.3. As Moise notes just before the quoted passage, the general idea of an isometry is a unifying the classical, more intuitive, notions of congruence for line segments, angles, triangles, circles and circular arcs. The first three are treated in the notes, and the the other two are defined on page 251 of Moïse. Proofs that the first three types of congruence are the same as isometry appear in the notes and on pages 252–261 of Moïse. In the remaining cases, it is also possible to prove that congruence is equivalent to isometry, but although the proofs are relatively straightforward they are also somewhat lengthy; if time permits, the details will be posted in another file.