*Classical superposition*

Coincidence [superposition] is either mere tautology**,** or something entirely empirical**,** which belongs **...** to external sensuous experience**.**

A. Schopenhauer (1788 – 1860)**,** ***The World as Will*(1818)**

The proofs of **SAS** and **SSS**  in the ***Elements*** are exceptional because they rely on a principle of moving an object without changing its size or shape**.** In both cases one starts with **△ABC** and **△DEF** such that **|∠ABC| = |∠DEF| ,**and the idea is to move so that side **[BA]** will lie on ray **[ED** and side **[BC]** will lie on ray **[EF;** by construction the vertex **B**  is sent to **E.** The goal is then to show that the images of **A** and **C**  are precisely the points **D** and **F,** so that we have a ***superposition*** of **△ABC**  directly on top of **△DEF,** and from this to conclude that the corresponding parts of the triangles have equal measurements**.** The following online sites contain several interactive videos illustrating the physical concept of superposition**:**

[**http://www.ies.co.jp/math/products/geo1/menu.html**](http://www.ies.co.jp/math/products/geo1/menu.html) (see the subheading**,**

***Congruent Figures and Triangles***)

[**http://standards.nctm.org/document/eexamples/chap6/6.4/index.htm**](http://standards.nctm.org/document/eexamples/chap6/6.4/index.htm)

[**http://standards.nctm.org/document/eexamples/chap6/6.4/part3.htm**](http://standards.nctm.org/document/eexamples/chap6/6.4/part3.htm)

[**http://standards.nctm.org/document/eexamples/chap6/6.4/part4.htm**](http://standards.nctm.org/document/eexamples/chap6/6.4/part4.htm)

Everyday experience with physical objects strongly suggests such rigid motions of objects are easy to achieve**.**  However**,** **we are dealing here with *mathematical objects* rather than *physical objects,*** so to be logically complete we must either deduce this somehow from our setting for geometry or else make an additional assumption to justify it**.** As noted before**,** there is speculation that Euclid may have been uncomfortable with the use of superposition**,** for there are numerous other places in the ***Elements*** where it could have been used equally well**.**

There is a more extensive discussion of the steps needed to formalize the concept of superposition in the file [**http://math.ucr.edu/~res/math133/superposition.pdf**](http://math.ucr.edu/~res/math133/superposition.pdf)**.**