Math 153
Spring 2021

Notes for the April 23 lecture

These notes are meant to sketch the portion of the lecture which was not recorded. The main reference for the material is the following document:

http://math.ucr.edu/~res/math153-2021/week04unit04y/history04Y.pdf

Burton only mentions Apollonius briefly.

Unit 4Y deals with Apollonius of Perga, another outstanding Greek mathematician from the Hellenistic period. A map showing the location of Perga appears in the online file **http://math.ucr.edu/~res/math153-2021/week04unit04y/Perga.png**. Apollonius is mainly known for his writings on ***Conics***, which consisted of 8 books. The first four were mainly a summary of earlier results, probably with some enhancements due to Apollonius himself, and the last four contained original discoveries that were extremely deep for their time. In fact, the last book has been lost for centuries and our only sources for Books 5 – 7 are Arabic translations of the original Greek manuscripts.

The curves called conic sections can be described in several different ways, each having its own advantages and disadvantages. The listing below follows the main reference mentioned above:

0. **Definition as intersections of planes with a right circular cone.**
1. **Definition using quadratic equations in coordinate geometry.**
2. **Definition using two focal points for the curve.**
3. **Definition using one focal point and directrix line for the curve.**

The main notes and **http://math.ucr.edu/~res/math153-2021/week04unit04y/history04d.pdf** contain references and details of the proofs that the four characterizations of conics are equivalent.

Of course, the circle is the best known and probably most studied of all the conics, largely due to its symmetry properties. One aspect of these is that every line passing through the center can be viewed as an axis of symmetry. In contrast, for each of the remaining conics there is a specific pair of perpendicular lines which play key roles in the curves' geometric structure. These lines are shown in the crude drawings of the following online files:

http://math.ucr.edu/~res/math153-2021/week04unit04y/Whiteboard.apr23-02.png

http://math.ucr.edu/~res/math153-2021/week04unit04y/Whiteboard.apr23-03.png

Nowadays we can discuss the geometric structure of conics using analytic geometry and calculus. However, before the time of Apollonius the Greek geometers had already discovered many of the concepts related to such curves. For example, they knew about the asymptotic lines for a hyperbola, and they also understood the notions of tangent lines for conics. Furthermore, they also understood that two distinct conics had at most 4 points in common. Comments on the last two points appear in the online files **http://math.ucr.edu/~res/math153-2021/week04unit04y/Whiteboard.apr23-04.png** and **http://math.ucr.edu/~res/math153-2021/week04unit04y/Whiteboard.apr23-04a.png**.

The final item discussed before the start of the lecture recording was a result from Book 4 of the ***Conics***. This result may be viewed as a geometric version of the standard formula for the slope of the tangent line to a parabola at a point which is not the vertex (where the curve intersects the axis of symmetry):

$$\text{If } f(x) = ax^2, \text{ then } f'(x) = 2ax.$$

A modern proof of Apollonius' result (using differential calculus) appears in the file **http://math.ucr.edu/~res/math153-2021/week04unit04y/history04a.pdf**.