One more example

Solve $x \equiv 9 \bmod 37$, $x \equiv 7 \bmod 23$

Note Since $37 \cdot 23 = 851$, there is exactly one solution x between 0 and 850.

FIRST Find a, b so $37a + 23b = 1$

Euclidean algorithm	Expression as an integral linear combination of 37 & 23
$37 - 23 = 14$	$37 - 23$
$23 - 14 = 9$	$23 - (37 - 23) = 2 \cdot 23 - 37$
$14 - 9 = 5$	$(37 - 23) - (2 \cdot 23 - 37) = 2 \cdot 37 - 3 \cdot 23$
$9 - 5 = 4$	$(2 \cdot 23 - 37) - (2 \cdot 37 - 3 \cdot 23) = 5 \cdot 23 - 3 \cdot 37$
$5 - 4 = 1$	$(2 \cdot 37 - 3 \cdot 23) - (5 \cdot 23 - 3 \cdot 37) = 5 \cdot 37 - 8 \cdot 23$

Bottom line $5 \cdot 37$ cong $1 \bmod 23$
$(-8) \cdot 23$ cong $1 \bmod 37$

DO ALGEBRA NOW. Write $x = 23k + 7$

Want $23k + 7 \equiv 9 \bmod 37$, or $23k \equiv 2 \bmod 37$

Apply $(-8) \cdot 23 \equiv 1 \bmod 37$. Get $k \equiv -16 \bmod 37$
or $21 \equiv -16$

Hence $k = 37m + 21$, so $x = 23(37m + 21) + 7$
which simplifies to $490 + 851m$.

Check $490 = 13 \cdot 37 + 9$
$490 = 23 \cdot 21 + 7$