SOLVE

$x \equiv 1 \ (2)$

$x \equiv 2 \ (5)$

$x \equiv 1 \ (7)$

① $x = 2n+1$ some n

But then $2n+1 \equiv 2 \ (5)$, so

$2n \equiv 1 \ (5)$. Since $2 \cdot 3 \equiv 1 \ (5)$ get

$n \equiv 3 \ (5)$, or

② $n = 5p+3$, $x = 10p+7$.

Now $10p+7 \equiv 1 \ (7)$ so

$3p \equiv 10p \equiv 1 \ (7)$ $3 \cdot 5 \equiv 1 \ (7)$, so

$p \equiv 5 \ (7)$ or

③ $p = 7q+5$, so that

$x = 10(7q+5)+7 = 70q+57$.

Check these values satisfy the original congruences.