Impedance

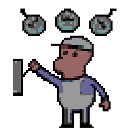
The main source for this material is the following online site:

http://regentsprep.org/REgents/mathb/2C3/electricalresouce.htm

Does anyone ever really use Complex Numbers?

Surprise! Complex numbers are not just a "math oddity" that only we crazy mathematicians enjoy.

Application to Electrical Engineering:



First, set the stage for the discussion and clarify some vocabulary. Information that expresses a single dimension, such as linear distance, is called a ***scalar*** quantity in mathematics. Scalar numbers are the kind of numbers students use most often. In relation to science, the voltage produced by a battery, the resistance of a piece of wire (ohms), and current through a wire (amps) are scalar quantities.

When electrical engineers analyzed alternating current circuits, they found that quantities of voltage, current and resistance (called ***impedance*** in AC) were not the familiar one-dimensional scalar quantities that are used when measuring DC circuits. These quantities which now alternate in direction and amplitude possess other dimensions (frequency and phase shift) that must be taken into account.

In order to analyze AC circuits using the approach which works so well for DC circuits, it is necessary to represent multidimensional quantities. In order to accomplish this task, scalar [real] numbers were abandoned and ***complex numbers*** were used to express the two dimensions of frequency and phase shift at one time.

In mathematics, the letter i is used to represent imaginary numbers. In the study of electricity and electronics, the letter j is used to represent imaginary numbers so that there is no confusion with **i**, which in electronics represents current. It is also customary for scientists to write the complex number in the form $\mathbf{a} + j\mathbf{b}$.

For AC circuits there is a modified Ohm's Law formula $\mathbf{E} = \mathbf{I} \cdot \mathbf{Z}$, where $\mathbf{E}$ is voltage, $\mathbf{I}$ is current, and $\mathbf{Z}$ is impedance.

Possible Student Questions:

- The impedance in one part of a series circuit is $\mathbf{2+j8}$ ohms, and the impedance in another part of the circuit is $\mathbf{4-j6}$ ohms. Find the total impedance in the circuit. **Answer:** $\mathbf{6+j2}$ **ohms**

- The voltage in a circuit is $45+j10$ volts and the impedance is $3+j4$ ohms. What is the current?

  Answer:

  $\mathbf{E = I \bullet Z}$

  $\mathbf{45 - j10 = I \bullet (3 + j4)}$

  $$\frac{45+j10}{3+j4} = I$$

  $$\frac{45+j10}{3+j4} \bullet \frac{3-j4}{3-j4} = I$$

  $$\frac{135-j180+j30-j^2 40}{9-j^2 16} = \frac{175-j150}{25} = 7-j6$$

  $\mathbf{7-j6}$ **amps**

- The current in a circuit is $4+j2$ amps and the impedance is $5-j$ ohms. What is the voltage?

  Answer:

  $\mathbf{E = I \bullet Z}$

  $\mathbf{E = (4 + j2) \bullet (5 - j)}$

  $$(4+j2)\bullet(5-j) = 20 - j4 + j10 - j^2 2$$

  $$= 22 + j6$$

  $\mathbf{22 + j6}$ **volts**

ADDED NOTE. **(RS)** The formulas for the total impedance in series and parallel AC circuits are formally the same as their counterparts for resistance in series and parallel DC circuits. In particular, this means that the computation of impedance for parallel circuits relies heavily on the standard formula for the reciprocal of a nonzero complex number: If $\mathbf{a+jb}$ is nonzero (in other words, at least one of **a** and **b** is nonzero), then its reciprocal is given by $\mathbf{(a-jb)/(a^2+b^2)}$. Note that the denominator of this expression is nonzero if at least one of **a** and **b** is nonzero.

Hopefully, this will help to answer the question, "Does anyone ever really use complex numbers?"