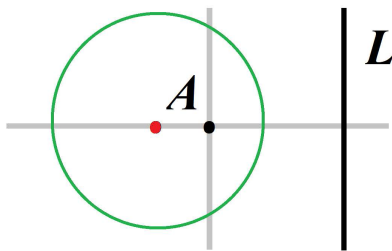# Another Locus Exercise

Problem.   Suppose we are in the coordinate plane, and let A and L denote the origin and the vertical line $x = 1$ respectively. Find the locus (set) of all points P such that the square of the square of the distance from P to A is equal to the distance from P to L.

Solution.   If $P = (x, y)$, then the square of the distance from P to A equals $x^2 + y^2$ and the distance from P to L equals $|x - 1|$. Therefore the defining equation of the set is $x^2 + y^2 = |x - 1|$. It will be convenient to split this into a pair of equations:

$$x^2 + y^2 = x - 1 \quad \text{or} \quad x^2 + y^2 = 1 - x$$

The first equation can be rewritten in the form

$$\left(x - \frac{1}{2}\right)^2 + y^2 = -\frac{3}{4}$$

and the second can be rewritten in the form

$$\left(x + \frac{1}{2}\right)^2 + y^2 = \frac{5}{4}.$$

There are no real number solutions to the first equation because a sum of real squares is never a negative real number, and the solutions to the second are given by the points on the circle of radius $\frac{1}{2}\sqrt{5}$ with center $(-\frac{1}{2}, 0)$. Therefore the locus is equal to this circle.∎