## How a slide rule works

The $\mathbf{C}$ and $\mathbf{D}$ scales on a slide rule uses base $\mathbf{1 0}$ logarithms, so if the length of the scale from the $\mathbf{1}$ on the left hand edge to the $\mathbf{1}$ on the right hand edge is $\mathbf{1}$ linear unit and $x$ is a number between $\mathbf{1}$ and 10 , then the mark corresponding to $x$ is $\log _{10} x$ units from the left hand edge.

(Source: http://web.mit.edu/2.972/www/reports/slide rule/slide rule.html)
To multiply $\mathbf{3}$ by $\mathbf{2}$, one slides the $\mathbf{C}$ scale so that the left hand edge meets the $\mathbf{D}$ scale at 3, and find the point on the $\mathbf{C}$ scale marked 2. Now look at the point on the $\mathbf{D}$ scale which lies underneath it and notice that this point is marked $\mathbf{6}$, which is $\mathbf{3}$ times $\mathbf{2}$.


## (Same source as before)

Why does this work? The left edge of the $\mathbf{C}$ scale is located $\log _{10} \mathbf{3}$ units away from the left edge of the $\mathbf{D}$ scale, and the point marked $\mathbf{2}$ on the $\mathbf{C}$ scale is another $\boldsymbol{\operatorname { l o g }}_{10} \mathbf{2}$ units away from the left edge of the $\mathbf{C}$ scale. The point underneath the latter on the $\mathbf{D}$ scale is then $\log _{10} \mathbf{3}+\log _{10} \mathbf{2}=\log _{10} \mathbf{6}$ units away from the left edge of the $\mathbf{D}$ scale. By construction, the marking on the $\mathbf{D}$ scale for this point is equal to $\mathbf{6}$.

A second possibility The preceding method works well for multiplying two digit numbers if their product is less than $\mathbf{1 0}$. Suppose now that the product is greater than $\mathbf{1 0}$; for example, consider the product $\mathbf{4}$ times 3 . If we try to carry out the procedure outlined above, we find that the point marked by 4 on the $\mathbf{C}$ scale will go beyond the markings on the D scale. In such cases, we first slide the $\mathbf{C}$ scale so that its right end sits over $\mathbf{4}$ on the $\mathbf{D}$ scale. Next, we locate the point marked by $\mathbf{3}$ on the $\mathbf{C}$ scale and look at the point on the $\mathbf{D}$ scale which lies underneath it. Consider the
position of the left end of the $\mathbf{C}$ scale with respect to the left end of the $\mathbf{D}$ scale. Since the right end of the $\mathbf{C}$ scale lies at the point which is $\log _{10} 4$ units to the right of the $\mathbf{D}$ scale's left end, the left end of the $\mathbf{C}$ scale lies at the point which is $\log _{10} \mathbf{4 - 1}$ units to the left of the $\mathbf{D}$ scale's left end. Therefore the point on the $\mathbf{C}$ scale which is $\log _{10} 3$ units to the right of that scale's left end will be

$$
\log _{10} 4+\log _{10} 3-1=\log _{10} 12-1=\log _{10} 1.2
$$

units to the right of the $\mathbf{D}$ scale's left end. All of this is illustrated in the drawing below.


