

NAME: \_\_\_\_\_

**Mathematics 153, Spring 2005, Final Examination**

*Point values are indicated in brackets.*

1. [30 points] Using the identity

$$\frac{1}{kn} = \frac{1}{k(n+1)} + \frac{1}{kn(n+1)}$$

give three different ways of writing  $1/12$  as a sum of unit fractions

$$\frac{1}{p} + \frac{1}{q}$$

with distinct denominators. [*Hint:* Factor 12 as a product of two smaller positive integers several ways.]

2. [30 points] Suppose  $a$  and  $b$  are positive integers such that  $x = a$  and  $y = b$  satisfy the Diophantine equation  $x^2 - 2y^2 = 1$  (one specific example is  $a = 3$  and  $b = 2$ , but this will not really be needed to work the problem). Prove that  $a^2 + 2b^2$  and  $2ab$  satisfy the same equation. [Hint: What is  $(a^2 - 2b^2)^2$ ?]

3. [20 points] Two of the following sequences of perfect squares are consecutive terms in an arithmetic progression and two are not. Determine which two satisfy the arithmetic progression condition and which two do not.

(i)  $1^2, 5^2, 7^2$

(ii)  $3^2, 9^2, 12^2$

(iii)  $5^2, 12^2, 13^2$

(iv)  $7^2, 13^2, 17^2$

4. [30 points] Find a change of variables  $y = x + a$  that will transform the cubic equation  $x^3 + 6x^2 + 11x + 6 = 0$  to an equation of the form  $y^3 + py + q = 0$ , and use this equation to find all the roots of  $p$ . [Hint: The new polynomial factors into linear polynomials very easily!]

5. [60 points] Circle the appropriate answer for each item.

(i) About how much time passed between the appearance of Leibniz' work and Cauchy's definition of a limit?

- A. 50 to 100 years
- B. 100 to 150 years

(ii) Who denoted derivatives by  $dy/dx$  and who used  $\dot{y}$ ?

- A. Leibniz used  $dy/dx$  and Newton used  $\dot{y}$ .
- B. Newton used  $dy/dx$  and Leibniz used  $\dot{y}$ .

(iii) Who was first mathematician known to recognize the inverse relation between differentiation and integration?

- A. Barrow
- B. Huygens
- C. Wallis

(iv) Who published the first tables of base 10 logarithms?

- A. Briggs
- B. Galileo
- C. Napier

(v) Which of the following was strongly influenced by the other's earlier work?

- A. Fermat
- B. Vieta

(vi) Which of the following was NOT involved in deriving or publishing the cubic or quartic formulas?

- A. Cardano
- B. Ferrari
- C. Tartaglia
- D. Torricelli

6. [30 points] For each of the topics listed below, match the name of a person who contributed significantly to that topic using the letter key indicated below. No name should be used more than once.

- \_\_\_ Consecutive squares in an arithmetic progression
- \_\_\_ 14<sup>th</sup> century contributor to mathematics
- \_\_\_ Integration of some trigonometric functions
- \_\_\_ Mathematical theory of perspective drawing
- \_\_\_ Popularization of decimals
- \_\_\_ Use of  $x$ ,  $y$ ,  $z$  for unknowns.

- A** : Alberti
- B** : Cavalieri
- C** : Descartes
- D** : Ferro
- E** : Fibonacci
- F** : Gregory
- G** : Oresme
- H** : Pascal
- I** : Regiomontanus
- J** : Stevin