Math 153
Spring 2016

UPDATED GENERAL INFORMATION — APRIL 22, 2016

Examination rescheduling

To confirm earlier verbal statements made in class this past week, the in-class examination originally scheduled for Wednesday, April 27, has been rescheduled for **Friday, April 29.**

Information and suggestions concerning the examination

The examination will will be about 70 per cent mathematical problems and 30 per cent historical or short answer with opportunities for extra credit on the latter. The material coverage will go through the discussion of Apollonius' work on conics in `history04Y.pdf`.

MATHEMATICAL PROBLEMS. The old examinations give some idea of what to expect; these involve the use of algebra, precalculus or calculus to study issues related to the mathematics of ancient Egyptian, Babylonian and Greek civilizations. At least some parts will be straightforward variations on material from older examinations. Some questions will involve inequalities, and for these problems it will be very helpful to remember the following rule: If a, b, c, d are positive real numbers, then

$$\frac{a}{b} < \frac{c}{d} \quad \Longleftrightarrow \quad ad < bc .$$

In addition to the problems from the old examinations, here are a few problems that are related to the current examination or were considered but not included:

1. Show that if p is a prime and $p \geq 11$, then $10p$ is not a perfect number. Similarly, if p and q are odd primes then pq is not a perfect number. [*Hint:* Why is $(p-1)(q-1) \geq 8$?]
2. If p and q are odd primes with $p \neq q$, show that the numbers p^2 and q^2 do not form an amicable pair.
3. Suppose that A, B, C, D (in that order) are the vertices of a "nice" quadrilateral which is inscribed in a circle. Then the theorem on intercepted arcs in the notes then implies that the vertex angles satisfy $|\angle ABC| + |\angle ADC| = 180^\circ = |\angle BAD| + |\angle BCD|$. On the other hand, the vertex angles of a parallelogram satisfy both $|\angle ABC| = |\angle ADC|$ and $|\angle BCD| = |\angle BAD|$. Using these, prove that if a parallelogram in inscribed in a circle, then it is a rectangle.
4. Let $\mathbf{C}$ be the unit circle in the coordinate plane defined by the equation $x^2+y^2 = 1$, let $m > 0$, and suppose that the point $(\frac{1}{2}, 0)$ lies on the line $y = mx + b$ (so we can solve for b in terms of m). Prove that the line and circle meet in two points, one of which has a positive second coordinate and the other of which has a negative second coordinate. — This exercise verifies a special case of a tacit assumption in Euclid's *Elements*: If a line L contains a point P which is inside a circle, then L meets the circle in two points, say X and Y, and P is between X and Y.

5. This exercise verifies a special case of Pasch's Theorem, which was also a tacit assumption in Euclid's *Elements*: Consider the isosceles right triangle in the coordinate plane whose vertices are $(0, 0)$, $(2, 2)$ and $(-2, 2)$ and let L be the line defined by the coordinate equation $x+2y = 3$, so that the point $(1, 1)$ lies on both L and on the edge of the triangle with endpoints $(0, 0)$ and $(2, 2)$. Prove that L also contains a point on the edge with endpoints $(2, 2)$ and $(-2, 2)$ by showing that the coordinates of the point $(u, 2)$ where L meets the line joining the latter (which is the horizontal line $y = 2$) satisfy the inequality $-2 < u < 2$.

In each of the last three problems, drawing a sketch of the figure is strongly recommended as a starting point.

HISTORICAL QUESTIONS. These will involve major mathematical contributions or advances in various cultures and some basic knowledge of the time sequence of important developments. For example, such a question might be to state which happened first: The discovery that $\sqrt{2}$ is irrational or Plato's principle stating that an unmarked straightedge and compass were the ideal tools for geometric constructions. All of the discoveries and developments will be taken from the notes in the online directory up to the coverage limit, but some may have occurred after the era of Greek mathematics.