Math 153
Spring 2016

UPDATED GENERAL INFORMATION — JUNE 6, 2016

The final examination

This is a reminder that the examination takes place on Thursday, June 9, from 11:30 A.M. to 2:30 P.M. in the usual meeting room for the lecture. The file `update08.153.s16.pdf` contains a considerable amount of further information.

Additional office hours on Monday, June 6

I will be in my office (Surge 221) from 1 to 3 P.M., and possibly a little longer on either end if necessary (please contact me in that case). Also, on Tuesday, June 7, I will be monitoring the final exam in my other course between 9 and 11 A.M., so if you have short questions that will take less than 15 minutes it should be possible for me to answer them at that time; the location of that exam is Surge 284.

Correction/clarification to the previous update

In Problem 9, the object is to prove that Γ defines a **circle** rather than an ellipse (yes, a circle can be viewed as a special type of an ellipse, but ...).

Comments on biconditional arguments

When one solves locus problems using coordinate geometry, the usual approach is to start by translating the hypotheses into algebraic equations and to proceed by manipulating these equations until we get equations which describe the conditions listed in the conclusion. As noted in an answer to one of the questions in class, the proofs of locus theorems are always biconditional: One needs to prove that A implies B and also that B implies A. Fortunately, most of the standard manipulations involving equations are reversible, so that the equation at step n is true if and only if the equation at the next step is true. This is because the following operations on equations are reversible processes:

> Adding, subtracting, or multiplying both sides of an equation by the same quantity, where the quantity must be nonzero for multiplication.
>
> Dividing both sides of an equation by the same quantity, provided the latter is nonzero (you need to know that the quantity is nonzero in order to proceed!).
>
> Replacing an expression on one side of an equation by something which is equal to that expression.

There are also rules regarding reversible processes involving inequalities, but they will not be relevant to the locus problems considered in this course. The moral is that *manipulations of the given types in locus problems are biconditional, so one does not have to be careful about the issue in such problems.*

A more advanced discussion of the issues (*not needed in this course, but included for the sake of completeness*). In elementary algebra, the main place that one encounters irreversible processes (aside from multiplication and division by zero) is in certain square root problems: If two quantities are equal, then their squares are equal, but if the squares are equal then one cannot necessarily conclude that the initial quantities are equal without some further information. For example, if we already know that the initial expressions are both nonnegative, then the step in question is reversible.

For the record, an example of the square root difficulty is discussed on pages 8 and 9 of `mathproofs.pdf`. The goal is to solve the equation $x - 3 = \sqrt{30 - 2x}$, where the radical sign denotes the nonnegative square root of a nonnegative number, and the first step in the solution is to square both sides. This shows that a solution to the original equation is also a solution to $(x-3)^2 = 30 - 2x$. The derivation proceeds to show that $x = 7$ or $x = -3$ are the two possibilies, but it turns out that the second value is not a solution. This extraneous solution arises because the squaring operation at the first step is not a reversible process, and consequently a solution to the second equation need not be a solution to the first one.