More Continued Fractions

$$\frac{5}{7} = x_0$$

$$y_0 = \frac{1}{x_0} = \frac{7}{5} \qquad x_0 = \frac{1}{y_0} = \frac{1}{\frac{7}{5}}$$

$$y_0 = \underset{n_1}{1} + \underset{x_1}{\frac{2}{5}} \qquad x_0 = \frac{1}{1+\frac{2}{5}}$$

$$y_1 = \frac{1}{x_1} = \frac{5}{2} \qquad x_0 = \frac{1}{1+\frac{1}{y_1}}$$

$$y_1 = \underset{n_2}{2} + \underset{x_2}{\frac{1}{2}} \qquad x_0 = \frac{1}{1+\frac{1}{2+\frac{1}{2}}}$$

$$y_2 = \frac{1}{x_2} = 2$$

$$y_2 = \underset{n_2}{2} + \underset{x_3}{0} \quad \text{fractional piece}$$

So the process stops here.

Handle $\frac{3}{7}$ similarly

$$\frac{3}{7} = x_0$$

$$y_0 = \frac{1}{x_0} = \frac{7}{3}$$

$$y_0 = 2 + \frac{1}{3}$$
$$n_1 \quad x_1$$

$$x_0 = \frac{1}{y_0} = \frac{1}{\left(\frac{7}{3}\right)} = \frac{1}{2 + \frac{1}{3}}$$

$$y_1 = \frac{1}{x_1} = 3$$

$$y_1 = 3 + 0 \Rightarrow x_0 = \text{as above.}$$
$$n_2 \quad x_2$$

↑ stop process here

The proof that the process stops after finitely many steps is given in history 04c.pdf

One more

$x_0 = \frac{9}{16}$

$y_0 = \frac{16}{9} = 1 + \frac{7}{16}$ $\qquad x_0 = \cfrac{1}{1+\frac{7}{16}}$

$n_1 \quad x_1$

$y_1 = \frac{1}{x_1} = \frac{16}{7} = 2 + \frac{2}{7}$ $\qquad x_0 = \cfrac{1}{1+\cfrac{1}{2+\frac{2}{7}}}$

$n_2 \quad x_2$

$y_2 = \frac{1}{x_2} = \frac{7}{2} = 3 + \frac{1}{2}$ $\qquad x_0 = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+1/2}}}$ ∎

$n_3 \quad x_3$