1.C. An Example of the False Position Method

Versions of the Method of False Position, which gives successive approximations converging to a solution for an equation of the form **f(x) = 0,** were known and used in ancient Egyptian and Babylonian mathematics. The objective here is to describe the level of sophistication needed to develop such procedures by using the method to find the unique real root of the cubic equation

$$x^3 + x + 1 = 0.$$

This method applies particularly well in situations where there are two points **a** and **b** such that **f(a)** and **f(b)** are nonzero with different signs and there is exactly one value of **r** between **a** and **b** such that **f(r) = 0.** For the given example we have **f(–1) < 0 < f(–½)** and the function **f** is strictly increasing (because its derivative is always positive). Much of this discussion is adapted from the following site;

http://www.math.buffalo.edu/~pitman/courses/mth437/na/node20.html

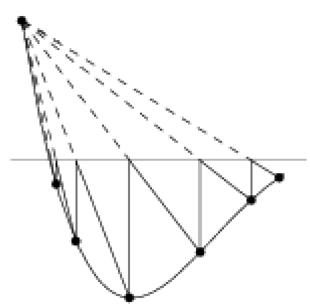
Suppose now that we are given a reasonable function **f** satisfying the conditions in the preceding paragraph. One naïve way to search for a root is to ***pretend that the function is linear between* a *and* b *and to find the value* c *for which this linear function's value is zero.*** In graphical terms, **c** is the *x* – intercept of the line between **(a, f(a))** and **(b, f(b)).** To find **c,** we first need to write out the equation for the line joining **(a, f(a))** and **(b, f(b)):**

$$y = f(b) + \left(\frac{f(b) - f(a)}{b - a}\right)(x - b)$$

If we set **y = 0** and solve for **x**, we obtain the *x* – intercept, which is **c:**

$$c = b - \left(\frac{b - a}{f(b) - f(a)}\right) f(b)$$

The next step is to evaluate **f(c)** and determine whether it is positive or negative or zero. If it is zero, then obviously we are done; otherwise, we let **c** replace either **a** or **b,** taking the one for which the sign of **f(c)** is equal to the sign of **f(a)** or **f(b).** The effect of this replacement is to narrow the search for the root to a slightly smaller interval. We can now repeat this procedure on the smaller interval, obtaining another, even smaller interval, and so forth. As suggested by the drawing below, in favorable cases the values for **c** obtained at the successive steps will converge to the root of the equation **f(x) = 0.**

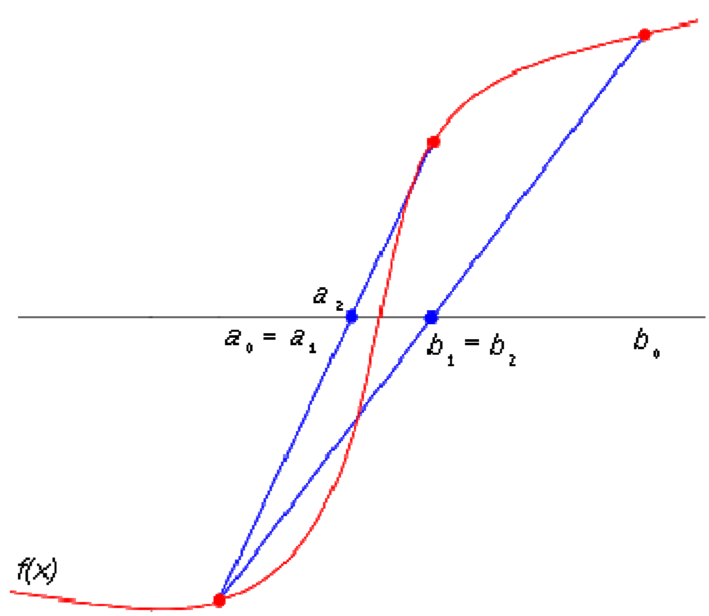


(***Source:*** **http://mathworld.wolfram.com/MethodofFalsePosition.html**)

Here is a table describing 15 iterations of the method for the cubic equation **$x^3 + x + 1 = 0$.** As indicated above, the initial interval is **[– 1, ½],** and we know that the values of the polynomial at the left and right end points are negative and positive respectively**.**

a	**b**	**f(a)**	**f(b)**	**c**	**f(c)**
-1	-0.5	-1	0.375	-0.636363636	0.105935387
-1	-0.636363636	-1	0.10593539	-0.671195652	0.026428288
-1	-0.671195652	-1	0.02642829	-0.679661646	0.006375484
-1	-0.679661646	-1	0.00637548	-0.68169102	0.001525358
-1	-0.68169102	-1	0.00152536	-0.682175816	0.000364224
-1	-0.682175816	-1	0.00036422	-0.682291533	8.6928E-05
-1	-0.682291533	-1	8.6928E-05	-0.682319148	2.07444E-05
-1	-0.682319148	-1	2.0744E-05	-0.682325738	4.9503E-06
-1	-0.682325738	-1	4.9503E-06	-0.682327311	1.1813E-06
-1	-0.682327311	-1	1.1813E-06	-0.682327686	2.81893E-07
-1	-0.682327686	-1	2.8189E-07	-0.682327776	6.72684E-08
-1	-0.682327776	-1	6.7268E-08	-0.682327797	1.60523E-08
-1	-0.682327797	-1	1.6052E-08	-0.682327802	3.83058E-09
-1	-0.682327802	-1	3.8306E-09	-0.682327803	9.14093E-10
-1	-0.682327803	-1	9.1409E-10	-0.682327804	2.18131E-10

In this example all the replacements involve the right hand endpoint**.** On the other hand**,** as suggested by the drawing below**,** there are cases in which successive replacements involve both right and left hand endpoints**.**



(***Source:*** **http://en.wikipedia.org/wiki/False_position_method**)

In addition to the references cited in Unit 1 of the notes, some additional information and comments about this method appear on pages 73 – 75 of the following standard textbook on numerical methods**:**

> **R. L. Burden and J. D. Faires,** *Numerical Analysis* (9th Ed.)**.** Brooks – Cole**,** Boston**,** 2011**. ISBN – 10:** 0 – 538 – 73351 – 9**.**

Also**,** here is an online reference which includes a proof that the method**'**s iterated approximations actually converges to the desired solution**:**

http://astro.temple.edu/~dhill001/course/numanalfall2009/False%20Position%20Section%202_2.pdf